German Economic Team Belarus

IPM Research Center

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Deseasonalizing Belarusian GDP

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Executive Summary

Various economic indicators have a seasonal pattern and demonstrate regular yearly fluctuations. Seasonal variation makes it difficult to analyze the raw data and does not allow identifying explicitly short-term dynamics of the indicator and its long-term trends. Therefore, to get an idea about the dynamics of an economic indicator without seasonal factor, different methods of seasonal adjustment are used. The quality of seasonal adjustment determines the readability of the results and final conclusions concerning the economic situation for policy making.

The paper proposes a novel approach that allows conducting seasonal adjustment when several observations in analyzed time series are contaminated and substantially distort the results of seasonal adjustment. A dramatic example of such a situation is seasonally adjusted data for Belarusian real GDP in 2013Q1, when an application of standard statistical methods resulted in an annualized growth rate of about 40%.

The proposed approach allows identifying the contaminated observations and carrying out seasonal adjustment of real GDP eliminating an influence of these observations. As a result, according to our estimates, the corrected figures for Belarusian real GDP growth in 2013Q1 were 0.8% and 3.2% for quarter-to-quarter and annualized growth rates, respectively. This approach can be used in any other situation when seasonal adjustment under data contaminations leads to the results which contradict the real economic situation.

The paper discusses in detail the methodology of seasonal adjustment of Belarusian real GDP and proposes policy recommendations that are of practical importance for statistical agencies, central banks, economic observers and academic researchers. This methodology and recommendations is taken into account by the National Bank while preparing of the revised seasonally adjusted quarterly growth rate of real GDP for the first half of 2013.

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Contents

1. Introduction

Various time series of economic indicators have a seasonal pattern and demonstrate yearly regular fluctuations. Seasonal variations make it difficult to analyze the raw data and do not allow getting explicitly an idea about short term dynamics of the indicator and its trend. Therefore, to analyze the dynamics of economic indicators without the influence of seasonal factor, different methods of seasonal adjustment are used. In practice of economic analysis, year-to year growth rates of corresponding indicator are often used. Such an approach is a very rough methods of seasonal adjustment ant it has a number of shortcomings. Statistical methods of seasonal adjustment provide more accurate results and they are widely used by the statistical agencies and central banks worldwide. Seasonally adjusted data are of great importance for relevant diagnostic of the economic situation in the short run, decision-making, informing economic agents and general public about economic environment, and economic analysis, econometric modeling and forecasting.

Currently, seasonal adjustment of economic data is not a widespread practice in Belarus. National Statistical Committee of the Republic of Belarus (Belstat) performs seasonal adjustment of real GDP and publishes seasonally adjusted data in the National Account Statistical Book (Belstat (2013)). Since these data are published with a two year lag (for instance, in 2013 seasonally adjusted data on real GDP are presented up to the end of 2011), the practical value of such an information is quite low. The National Bank of the Republic of Belarus (NBB) presents seasonally adjusted data for real GDP in their analytical review of the Main Trends in the Economy and Monetary Sphere (NBB (2013b)) and provides the data for the latest quarter available and the graph for annualized growth rate of real GDP. It is without doubt very important for an assessment of current economic situation. The International Monetary Fund also provides figures for seasonally adjusted GDP growth rates in Country Reports on Belarus (IMF (2013)).

The main motivation of the paper was a publication of real GDP at annualized growth rate for 2013Q1 in a recent analytical review of the NBB. In accordance with the presented data, annualized growth rate of real GDP was 37% that is explained in the commentary by the low base effect of 2012Q4 and changes in tax legislation (NBB (2013b)). Similar results were presented by the IMF. In the corresponding figure in the Country Report one can see that quarter-to quarter growth rate of real GDP in 2013Q1 is above 8%. At annualized rate it will be equal approximately to the same 37%. The IMF report remarks about the acceleration of economic growth the beginning of 2013 (IMF (2013)).

These growth rates of Belarusian real GDP do not correspond to both, common sense and real economic situation in the country. It should be noted, however, that in this case the standard statistical methods of seasonal adjustment are applied. In particular, the NBB made seasonal adjustment using standard procedure TRAMO/SEATS. Seasonal adjustment was conducted for the period 2000Q1–2012Q2, and then the seasonal factors obtained for this time span were used to produce seasonally adjusted data for the remaining quarters. We were able to replicate the IMF results quite accurately applying a well-known methods of seasonal adjustment based on X-11 seasonal filter without any additional settings. Why was the growth rate of real GDP in 2013Q1 so high?

It is known that the quality of seasonal adjustment depends on the quality of real GDP raw data, and statistical methods used. However, modern statistical software for seasonal adjustment makes it possible to obtain in many cases quite acceptable results using appropriate methods virtually automatically (worldwide practice of statistical agencies on unification of seasonal adjustment procedures and introducing standard software is to large extend based on this fact).

This led us to the following working hypothesis: the raw data of real DGP is possibly contaminated[1], at least for 2012Q4 and 2013Q1. The usage of the standard statistical method for seasonal adjustment in the presence of contaminated data can lead to inadequate results of seasonal adjustment, especially at the end of the time series. In the paper we attempted to resolve this problem by identifying contaminated observations using econometric techniques,

[1] Under contaminated data we mean the errors or inaccuracy in the data that distort the historical seasonal pattern in time series.

and subsequent corrections of seasonally adjusted time series. The proposed approach permits to obtain results that are consistent with real economic situation and to return seasonally adjusted time series to the "right" trajectory. It should be noted, that our aim is not to reevaluate official statistical data on real GDP. Our approach is an econometric one and it is based on available official data. Nevertheless, the problems in seasonal adjustment of the real GDP data and inadequate results for 2013Q1, derived from applying standard statistical methods, in our view, can indicate the possible statistical problems with original data.

Thus, the main objective of the paper is to propose an approach for seasonal adjustment of economic indicators under data contaminations in time series used. At the same time, we have attempted to resolve the problem of incredibly high growth rates of Belarusian GDP in 2013Q1 in the framework of econometric analysis and conventional methods of seasonal adjustment. The results are practically useful for statistical agencies, central banks, economic analysts and academic researchers.

The paper is organized as follows. The second section provides a general overview of seasonal adjustment; then briefly discusses the decomposition of the original data on its main components and stresses the important role of irregular component while calculating seasonally adjusted growth rates, especially in the annualized form; and finally considers the standard methods for seasonal adjustment and appropriate software. The third section analyzes the dynamic characteristics of real GDP, discusses the problems of seasonal adjustment arising from contaminated observations, and offers a novel approach for extreme observations identification based on impulse indicator saturated regression for irregular component. The fourth section provides a comparison of two commonly used methods of seasonal adjustment (X-12ARIMA and TRAMO/SEATS) while analyzing Belarusian real GDP, and presents revised seasonally adjusted real GDP growth rates. Additionally, the influence of a time span on the results of seasonal adjustment is also examined. The fifth section contains conclusions and policy recommendations.

2. Seasonal adjustment: General overview

2.1. What is seasonal adjustment?

Statistical data is widely used by officials and economic agents for decision-making and forecasting. Whereby, it is necessary to identify the important features of dynamics of economic indicators such as short term changes, mid- and long term trend, turning points in the dynamics and they behavior related to other indicators of interest. Seasonal patterns in the data make such an analysis difficult, because intrinsic dynamics of the economic indicator in this case is hidden from immediate observation. This problem is resolved by means of seasonal adjustment of economic time series, which can be defined as the processes of seasonal effects estimation and removing them from the original data. In other words, seasonal adjustment is the process of data simplification without the losses of important information contained in the raw data, so that it could be easier to interpret them (Bell, Hillmer (1984)).

To remove seasonal effect from the data, year-to-year growth rates are often applied. This approach is a simple and with that a rough seasonal adjustment. However, it has some serious shortcomings. First, although this approach permits to reduce constant annual seasonal effect, the complete elimination of seasonality is not always possible, as seasonal factor may gradually change within a year. Secondly, it cannot take into account the other calendar-related effects (e.g. the working day, moving holidays), and changes of the seasonal factor associated with changes in level of economic indicator (seasonal factor is assumed to be constant over the period). Thirdly, this method is sensitive to the various random effects. For instance, if an economic indicator in the previous period significantly decreased, as a result of an unexpected shock, then in the corresponding quarter of the following year one can observe an unusually large rate of growth (so-called base effect). Fourthly, the usage of year-to-year growth rates can lead to incorrect determination of the turning points in the dynamics of an economic indicator, as it does not take into account the quarterly changes within a year. This approach, however, can be considered as the starting point in analysis of the data with seasonal patterns.

The usage of statistical methods of seasonal adjustment enables to avoid the above mentioned shortcomings and to decompose the original time series into its unobservable components: seasonal component, trend (cycle) component, and irregular component. Then the seasonal factor is used for getting seasonal adjusted time series without any seasonal variation. It is important to note, that after seasonal adjustment only seasonal variation is removed from the data. As a result, the seasonally adjusted time series include, alongside with trend, random fluctuations, outliers and structural breaks caused by the various external and internal shocks. Seasonally adjusted data enable to make the correct calculations on the growth rate in the current period, to analyze the dynamics of an indicator in comparison with other economic variables, and to forecasts in the short-run.

2.2. Decomposition of the original data: Trend, seasonal factor and irregular

In this section we briefly consider the decomposition of time series into its unobservable components and discus several crucial points for the main task of the paper. Let Y_t is original observations of time series at time t, which can be represented as a product of three unobservable components, namely seasonal component (S_t), trend component (T_t), and irregular (random) component (I_t)[2]:

$$Y_t = S_t \cdot T_t \cdot I_t. \tag{1}$$

Now consider each of these components separately.

Seasonal components involves immediately the stable influence of seasonal factor, and also other systematic calendar effect, which are not stable within the year (for instance, working day effect, moving holidays effect). Trend component represent the general tendency and cyclical fluctuations of the time series in the mid- and long-run period. Irregular (random) component combines all effects that have not been taken into account by seasonal and trend components (outliers, characterized the extreme observations and data errors, extraordinary events). It is supposed that irregular is a stochastic variable with symmetrical distribution around the mean which is equal to one for multiplicative decomposition of time series into unobservebles.

In accordance with (1), seasonal adjustment of original data conducted as follows:

$$Y_t^{sa} = Y_t / S_t \ , \tag{2}$$

where Y_t^{sa} is a seasonally adjusted value of time series at time t.

Seasonally adjusted data are used for calculations of the period-to-period indexes i_y^{sa} and growth rate ($i_y^{sa} - 1$):

$$i_y^{sa} = Y_t^{sa} / Y_{t-1}^{sa} \ . \tag{3}$$

Additionally, seasonally adjusted data can be used for computation of annualized indexes which demonstrate what the growth would be if the current period rate continued for all period of the year (index is raised to power of n, equals the periodicity of the data):

$$(i_y^{sa})^n = (Y_t^{sa} / Y_{t-1}^{sa})^n \ . \tag{4}$$

It should be noted, that dynamics of seasonally adjusted data depends on two factors, namely changes of trend and irregular that immediately follows from (2) and (1). Since $Y_t^{sa} = Y_t / S_t = T_t \cdot I_t$, the expressions (3) and (4) can be represented as follows:

$$i_y^{sa} = \ T_t \cdot I_t / T_{t-1} \cdot I_{t-1} \ , \tag{5a}$$

$$(i_y^{sa})^n = T_t^n \cdot I_t^n / T_{t-1}^n \cdot I_{t-1}^n \ . \tag{5b}$$

Thus, it follows from (5a) and (5b) that if the influence of irregular component is quite strong, the growth rates calculated on the basis of seasonally adjusted data reflect not so much the systematic factors behind the dynamics of an indicator, as it changes due to random. The problem is amplified while using annualized growth rates, sine irregular component in this case is raised to power of four or twelve for quarterly or monthly data, respectively. This peculiarity of

[2] In the paper we consider only multiplicative decomposition of the time series into unobservable components. There is also additive method of decomposition (original data equals the sum of the components), pseudo-additive and log additive methods of decomposition. Adoption of one or another method of decomposition depends on characteristics of the data used. For seasonally adjusted Belarusian real GDP, multiplicative decomposition is applied. Standard software for seasonal adjustment allows choosing the decomposition method in automatic manner.

the annualized rates of growth should be taken into account when applying and interpreting the annualized growth rates. Moreover, an irregular component and its influence on seasonally adjusted data should be a subject of a careful analysis when calculating appropriate growth rates.

The relationship between seasonally adjusted time series, trend and irregular can be presented as follows:

$$T_t = Y_t^{sa} / I_t \, . \tag{6}$$

The expressions (1)–(6), mentioned above, will be taken into consideration further in conducting seasonal adjustment of Belarusian real GDP under possible data contaminations.

2.3. Standard methods of seasonal adjustment

The analysis of seasonality and appropriate seasonal adjustment is usually performed by of-the-shelf software. Currently, the two most popular methods of seasonal adjustment are used by statistical agencies and central banks, namely X-12ARIMA[3] from the US Census Bureau and TRAMO/SEATS[4] from the Bank of Spain. These two methods are considered in methodological recommendations of Eurostat as the main ones recommended for seasonal adjustment. (EES (2009)). Presently, the US Census Bureau offers new software X-13ARIMA-SEATS[5], which combines the capabilities of two mentioned method of seasonal adjustment, and makes it possible to compare them.

In the paper we utilized X-13ARIMA-SEATS software, allowing for comparison of the methods and models used. Although X-12ARIMA and TRAMO/SEATS methods of seasonal adjustment are differ significantly methodologically, they are based on the ARIMA (autoregressive integrated moving average) model of the following form: ARIMA(p, d, q)(P, D, Q), where p is the number of AR parameters, d is the order of integration, q is the number of moving average parameters, P is number of seasonal autoregressive parameters, D is the order of seasonal integration, Q is the number of seasonal moving average parameters. Determination of the parameters of the ARIMA model can be done either automatically or manually.

Now we briefly consider the features of X-12ARIMA and TRAMO/SEATS algorithms. X-12ARIMA is an iterative procedure. At first, it determines the trend and cycle component in the first approximation. Then, trend and cycle is removed from the original data. On the basis of the detrended data, seasonal component is derived for each period using the moving average filters. After that, the irregular component is determined by removing the seasonal component from the detrended data. The irregular is used to identify the extreme observations (outliers). As a result, the preliminary seasonally adjusted data is calculated by dividing the original data on the seasonal component, adjusted for extreme observations. Within the X-12ARIMA this procedure is repeated several times, until a final decomposition of the original time series into its unobserved components is achieved.

TRAMO/SEATS method consists of two consecutive procedures. In the first step TRAMO (Time series regression with ARIMA noise, missing observations, and outliers) provide automatic ARIMA modeling. In the second step SEATS (Signal Extraction in ARIMA Time Series) decomposes the time series into unobserved components using the signal extraction technique based on ARIMA model.

X-13ARIMA-SEATS software allows getting the diagnostics of the obtained models for seasonal adjustment using common criteria for assessing the quality of models. In particular, the appropriate tests for presence of seasonality, absence of residual seasonality in the seasonally adjusted data, absence of anomalies in residuals distribution for ARIMA model, stability of the seasonal adjustment procedure are applied. Eventually, the model with the best diagnostics is selected.

Finally, we would like to note that the seasonal adjustment using X-13ARIMA-SEATS can be accomplished by a variety of available software. For instance, the procedure X-13ARIMA-SEATS

[3] For details see http://www.census.gov/srd/www/x12a.
[4] For details see http://www.bde.es/webbde/en/secciones/servicio/software/econom.html.
[5] For details see http://www.census.gov/srd/www/x13as.

is included in the latest version of the econometric package Eviews (version 8)[6]. Apart from the original software of the U.S. Census Bureau, a similar approach is implemented in a test version of JDemetra+[7] from Eurostat. All these software packages allow achieving almost identical results of seasonal adjustment. In this paper we use the original software X-13ARIMA-SEATS from the U.S. Census Bureau, and also Eviews 8 and JDemetra+ as an additional tool while choosing the modes and assessing their quality.

3. Seasonal adjustment of Belarusian real GDP under possible data contamination

3.1. Data used and its characteristics

In the paper we use the quarterly data of real GDP in average 2009 prices for the period 1995Q–2013Q2[8]. We utilize the longest available time span, since such data are often used for econometric modeling and forecasting of Belarusian economy. Applying statistical methods for seasonal adjustment, it is desirable to use fairly long time series (7-10 years or more). Nevertheless, the usage of the longest available time series is not always appropriate because of possible changes in the methodology and comparability of the data, significant structural breaks, etc. Therefore, in addition to the maximal data span (1995Q–2013Q2, 74 quarters) a shorter period (2002Q1–2013Q2, 46 quarters) is also considered.

It should be noted, that official statistics do not represent the real GDP data in average 2009 prices for the considered period. Thus, the real GDP data for a number of years in 1995, 2000 and 2005 prices were converted into real GDP in 2009 prices by using available quarterly growth rates of real GDP. We understand some conventionality of such an approach, however, in our view the resulting data, reflect quite well the dynamics and seasonal patterns of real GDP over the period.

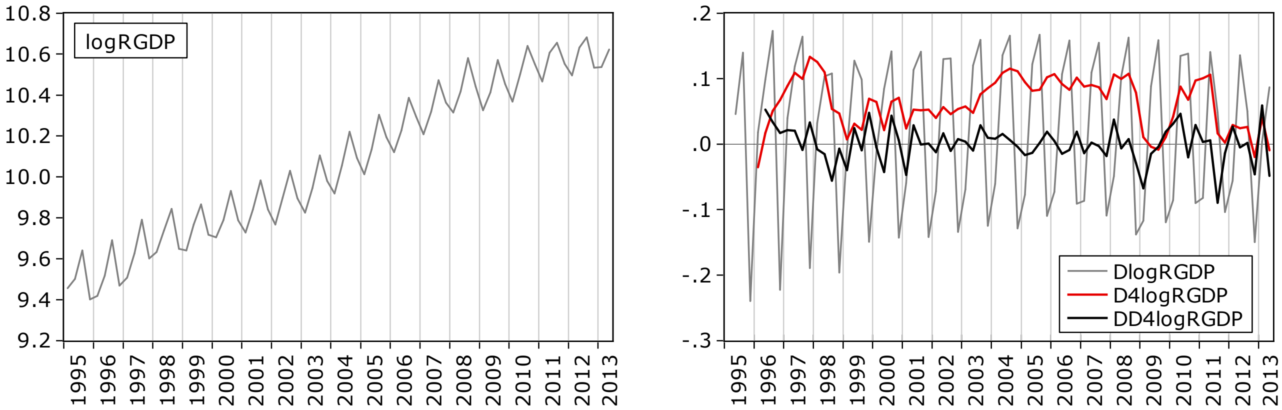
To obtain seasonally adjusted data of high quality, it is necessary to specify correctly the model ARIMA(p, d, q)(P, D, Q). In particular, the determination of the order of integration of the data is of great importance, since the statistical methods of decomposition of the time series into the unobservable components are applied only to the stationary data. When the data is expressed in logarithmic form, then if non-stationary variable becomes stationary by taken the first differences, it is said that variable is integrated with the order equal to 1, or I(1). In this case, it is assumed that the original time series contains a unit root. Accordingly, the first differences are stationary variable with a null order of integration, I(0). Such characteristics of the data can be checked using appropriate econometric tests. When the data is seasonal, the problem is complicated because the data may contain both regular unit root and seasonal unit roots that requires the usage of seasonal differences. Put it differently, seasonal adjustment with ARIMA (p, d, q)(P, D, Q) raises the question of whether it will be enough to use the only seasonal differences to make the original time series stationary, or additionally the regular differences are needed.

Within X-13ARIMA-SEATS the choice of d and D can be done automatically. However, as our experience shows, the order of integration of data determined in the automatic mode is sometimes not consistent with the actual dynamic characteristics of the time series.

Figure 1 shows the different aspects of real GDP dynamics in Belarus for the period 1995Q1-2013Q2. The time series of original data is expressed in natural logarithms. This is a convenient way of representing data when testing for a unit root, as the first logarithmic differences in our case are approximations of quarterly growth rates of real GDP, and the fourth logarithmic difference approximates its rate of growth to the corresponding quarter of the previous year. As follows from Figure 1, real GDP (logRGDP) has an upward trend and a pronounced seasonality. The first differences (DlogRGDP) also reflect seasonal fluctuations of real GDP, which hides its true dynamics. Seasonal differences of real GDP (D4logRGDP) eliminate seasonality and give some idea of its dynamics.

[6] For details see http://www.eviews.com/EViews8/ev8ecx13_n.html.
[7] For details see http://www.cros-portal.eu/content/jdemetra.
[8] All raw data used in this paper is presented in Annex A, Table A1.

Figure 1: Dynamic characteristics of real GDP



Note. $\log RGDP$ stands for logarithm of real GDP; $\Delta\log RGDP = \log RGDP_t - \log RGDP_{t-1}$; $\Delta_4 \log RGDP = \log RGDP_t - \log RGDP_{t-4}$; $\Delta\Delta_4 \log RGDP = \Delta_4 \log RGDP_t - \Delta_4 \log RGDP_{t-1}$; D is the difference operator.

Source: own calculations based on Belstat data.

Table 1: HEGY seasonal unit root test

Variable	Model specification		H_0	Test	Test statistics	Critical values	
	Deterministic terms	Number of lags				5%	1%
Log RGDP	Constant, Trend, Seasonals	1	$H_0: \pi_1 = 0$	$t_{\pi1}$	−1.47	−3.96	−3.39
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−2.19	−3.41	−2.82
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	1.93	8.76	6.55
	Constant, Trend	1	$H_0: \pi_1 = 0$	$t_{\pi1}$	−1.46	−3.98	−3.40
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−2.03	−2.53	−1.93
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	0.54	4.76	3.05
	Constant, Seasonals	1	$H_0: \pi_1 = 0$	$t_{\pi1}$	−1.64	−3.41	−2.84
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−2.18	−3.41	−2.83
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	1.91	8.79	6.57
	Constant	1	$H_0: \pi_1 = 0$	$t_{\pi1}$	−1.82	−3.42	−2.85
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−2.02	−2.53	−1.93
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	0.57	4.83	3.08
Δ_4Log RGDP	Constant, Seasonals	0	$H_0: \pi_1 = 0$	$t_{\pi1}$	−3.04	−3.41	−2.84
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−3.76	−3.41	−2.83
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	35.71	8.79	6.57
	Constant	0	$H_0: \pi_1 = 0$	$t_{\pi1}$	−3.08	−3.42	−2.85
			$H_0: \pi_2 = 0$	$t_{\pi2}$	−3.74	−2.53	−1.93
			$H_0: \pi_2 = \pi_3 = 0$	F_{34}	38.21	4.83	3.08

Note. The following regression is used to conduct HEGY seasonal unit root test:

$$\Delta_4 y_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-1} + \pi_4 z_{3,t-2} + \sum_{j=1}^{p-4} \alpha_i \Delta_4 y_{t-j} + \epsilon_t \ ,$$

where $\Delta_4 y_t = (1 - L^4) = y_t - y_{t-4}$; $z_{1,t} = (1 + L + L^2 + L^3) y_t$; $z_{2,t} = (-1 + L + L^2 + L^3) y_t$; $z_{3,t} = -(1 - L^2) y_t$ with L being the lag operator; ϵ_t are residuals. The number of lagged seasonal differences is chosen so to eliminate residual autocorrelation. The null hypotheses $H_0: \pi_1 = 0$, $H_0: \pi_2 = 0$ and $H_0: \pi_2 = \pi_3 = 0$ corresponds to test for regular, semiannual and annual unit root, respectively. The above mentioned regression is estimated by OLS and the hypotheses are tested using corresponding t-test for the first two hypotheses ($t_{\pi1}$, $t_{\pi2}$) and F-test for the third one (F_{34}). The critical values are taken from Franses, Hobijn (1997). JMulti econometric software is used for calculations[9]. Rejections of the null hypotheses are marked in gray.
Source: own estimations.

[9] For detail see http://www.jmulti.com.

However, the season differences do not seem stationary. Perhaps, to obtain stationarity the first differences of seasonal differences are also needed. This series (DD4logRGDP) fluctuates around zero mean and visually looks stationary. Since the time series of real GDP can contain both seasonal and regular unit roots, we applied HEGY-test for seasonal unit root (Hylleberg, et al. (1990)). This test is allowed to determine the presence (absence) of regular unit root and seasonal unit roots at the semi-annual and annual frequencies in the analyzed data. We conducted HEGY-test both for the log level of real GDP and for its seasonal differences. The null hypothesis of a unit root is rejected, if the actual values of the tests exceed the critical value at an appropriate significance level (5% or 1%).

As follows from Table 1, for real GDP the null hypothesis of regular unit root and seasonal unit roots is not rejected for any of examined test specifications. Consequently, real GDP is a non-stationary variable having both regular and seasonal (semiannual and annual) unit roots. The usage of seasonal differences eliminates seasonality and correspondently the seasonal unit roots. Herewith, the hypothesis of a regular unit root is rejected at the 5% significance level, but not rejected at the 1% level. As can be seen from Figure 1, the dynamics of seasonal differences demonstrates several shifts of the mean. For this reason, in addition to HEGY-test we used ADFGLS-test (Elliot, et al. (1996)), where original data is demeaned by using a generalized least squares and the demeaned data are used for unit root testing. The results of this test support the view that seasonal differences are not a stationary variable (ADFGLS-test is equal -1.76, with critical values equal to -1.95 and -2.60 at 5% and 1% significance levels, respectively).

Thus, the visual representation of data and a formal econometric analysis suggests that the real GDP is non-stationary variable and contains the regular and seasonal unit roots. Seasonal differences, most likely, do not make the time series stationary. To ensure the stationary one has to use also the first difference. Based on these results, we apply both an automatic selection of ARIMA(p, d, q)(P, D, Q) and semi-automatic selection fixing parameters d=1 and D= 1, while performing seasonal adjustment.

3.2. Why is seasonally adjusted growth rate for 2013Q1 incredibly high?

As mentioned earlier, application of the standard methods of seasonal adjustment leads to an annualized growth rate of GDP in 2013Q1 equal to 37%. If one use the data for 1995Q1-2013Q1, such a result can be accurately reproduced by using well-known seasonal filter X-11 without ARIMA model. Addition of the preliminary data for 2013Q2 (just one observation) even increases the growth rate of real GDP in 2013Q1. Let's consider this issue in more details. Figure 2 shows the results of the decomposition of real GDP into unobservable components over the period 1995Q1-2013Q2 using the X-11 filter and then adjusting for seasonality. In Figure 2 a conventional notations of the variables (D11, D12, D13) for X-13ARIMA-SEATS and similar software for seasonal adjustment are used.

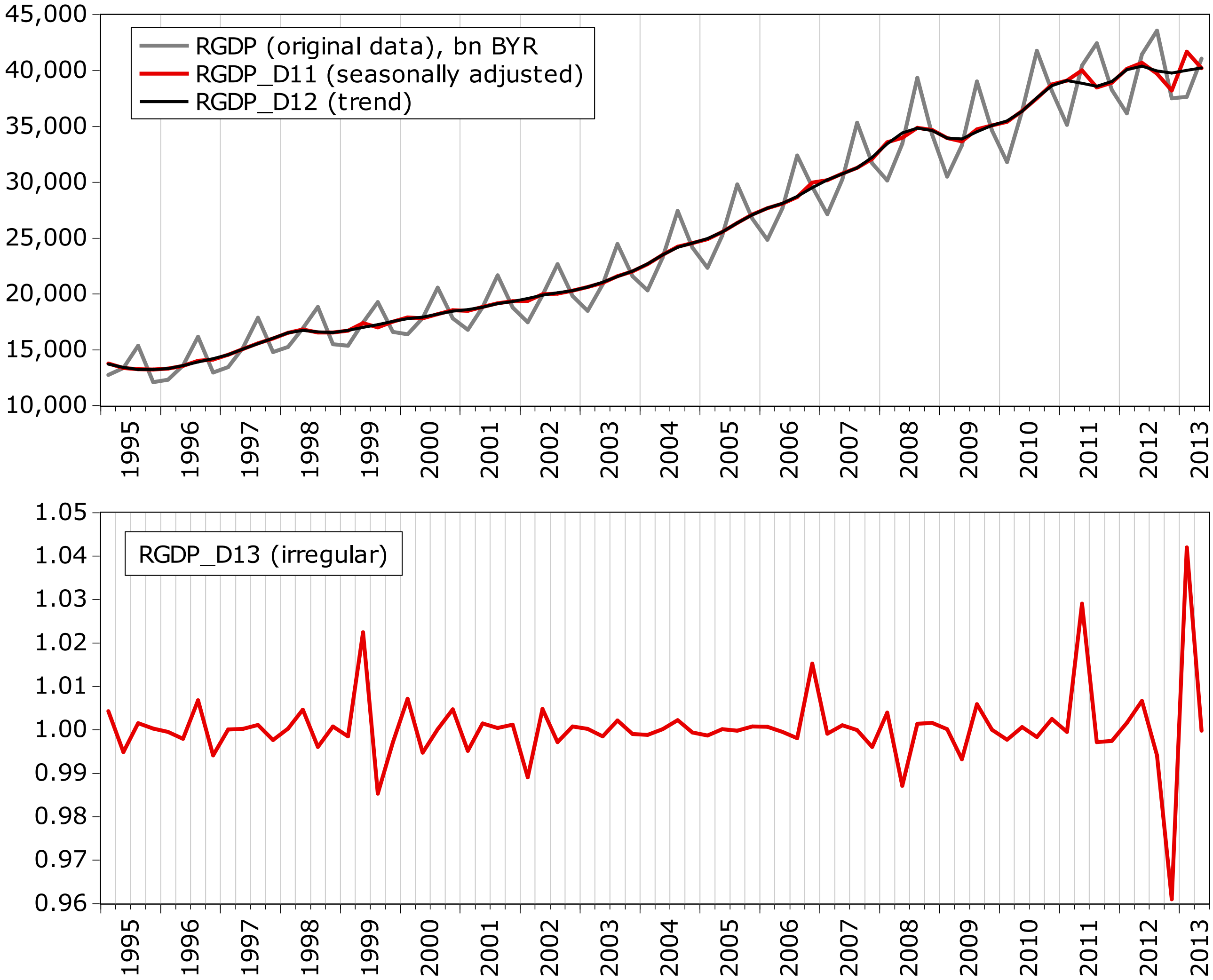
According to the obtained results, the growth of real GDP in 2013Q1 is equal to 1.091, or 9.1%, and in the annualized format — 1.418 or 41.8%, respectively. If we look at the graph of the original real GDP data, it is easy to see that in 2012Q4 and 2013Q1 there is an unexpected short term alteration of historical seasonal pattern. Namely this alteration has had a negative impact on the quality of the seasonal adjustment in the current period. Now let's consider the behavior of the irregular component. As one can see, it demonstrates very large deviations from its mean equal to one (0.961 in 2012 and 1.042 in 2013).

The graph of irregular component has several other significant outliers, but two of them mentioned above are the largest ones and they are placed at the end of time series. According to (5a), these outliers have a very strong influence on the results of seasonal adjustment for these dates. Since trend component, in accordance with (6), is not affected by the irregular, in this case it more accurately reflects the dynamics of real GDP (1.007 or 0.7%, and in annualized values 1.028 or 2.8%, respectively).

Thus, the observations for 2012Q4 and 2013Q1 that are unrepresentative for historical seasonal pattern, is the main reason of artifact in seasonal adjustment of real GDP at the end of the time series. As follows from (5b), an application of annualized growth rate substantially aggravates the problem.

Since there is no satisfactory explanation of the situation with original data of real GDP, we consider this problem in the context of seasonal adjustment as a consequence of data contaminations, and further propose possible solutions of the issue.

Figure 2: Decomposition of real GDP using X-11 filter



Source: own estimations.

3.3. Detection of contaminated data and correction of seasonally adjusted series

Generally, outliers in irregular component reflect extraordinary events and, if they have the proper economic interpretation, should be included into seasonally adjusted data. In the case of data contaminations we have to fix the inverse problem, namely to identify abnormal observations and eliminate their influence from seasonally adjusted data.

It should be noted, that within X-11 and X-12ARIMA, along with the extraction of seasonal factor, trend and irregular and obtaining appropriate seasonally adjusted data, there are a number of useful auxiliary data in the software output. In our case table C17 is of particular interest, which presents the final weight for irregular component. Normal observation is assigned a weight of 100, and all sorts of outliers take a value less than 100, up to 0 for the most significant extreme values. Table C17 from seasonal adjustment output shows that the analyzed data has 11 such outliers; six is them have a weight equal to 0. The results from table C17 are used in other auxiliary table E2, which contains modified seasonally adjusted data[10]. These data are seasonally adjusted time series (D11), wherein the observation with the weights of irregular component in table C17 are equal to zero, are replaced by corresponding values of trend (D12). According to table C17, the final weights of irregular for 2012Q4 and 2013Q1 are equal

[10] These tables from X-11 output are presented in Annex D.

to 0, i.e. they are significant outliers. All other outliers do not have a substantial impact on the seasonal adjustment in the current period.

Such a method of identification of the outliers is available only in X-12ARIMA. To resolve the problem with contaminated data, we need a more general approach that corresponded to table C17 in X-12ARIMA, and at the same time would be applicable for the analysis of irregular component while using TRAMO/SEATS method of seasonal adjustment.

For this purpose, we propose to use the method of impulse indicator saturation (IIS), which allows identifying the structural breaks, outliers and data contaminations. The theory of impulse indicator saturation is discuses in Hendry, et al. (2008); Johansen, Nielsen (2009). This method refers to the robust statistical methods, which tend to eliminate the possible data contaminations and solve at the same time the problem of so-called "fat tails" in the distribution by eliminating extreme observations. Some examples of empirical application of impulse indicator saturation are considered in Hendry, Mizon (2011); Castle, et al. (2012).

The essence of this method applying to our problem is as follows. For irregular components (I_t), deriving from the decomposition of time series, the following regression is used:

$$I_t = \mu + \sum_{s=1}^{T} \delta_s D_{s,t} + \epsilon_t, \tag{7}$$

where μ is a constant; $D_{s,t}$ is impulse indicator variable equal to 1 for period$s$, and 0 otherwise; δ_s is the coefficient at impulse indicator variable; regression residuals is $\epsilon_t \sim iid(0, \sigma^2)$; $t = 1, \ldots, T$.

As follows from (7), impulse indicator variable is created for each observation that takes a value equal to 1 for particular period and the values of 0 for all other periods. Since the number of impulse indicators is equal to the number of observations, in the case with a constant, inclusion of all of them in the regression is not possible, because the number of variables (impulse indicator and a constant) exceeds the number of observations. However, impulse indicators can be included in the model as a separate block. *Autometrics* routine (Doornik (2009)) used in the econometric package *OxMetrics* performs optimal partition of the sample on any number of blocks while selecting the final model and allows to identify statistically significant impulse indicator variables for an appropriate level of significance. An application of impulse indicator saturation techniques for the analysis of irregular component from time series decomposition permits to determine and eliminate the adverse effect of contaminated data.

The results of impulse indicator technique application for identification of the extreme values based on regression (7) are shown graphically in Figure 3[11]. The regression (7) is estimated at the different significance levels ($\alpha = 1\%$ and $\alpha = 0.01\%$). The tighter the significance level, the lower the probability that observation will be selected as extreme value, when actually it is not the fact. In Figure 3 extreme observation are depicted as the deviations from the mean equal to 1. Additionally, in Figure 3 the final weights for irregular components from table C17 are presented for comparison. In this case the extreme values are depicted as deviations from 1 up to 0.

Impulse indicator saturation methods with significance level $\alpha = 1\%$ detected 11 extreme observations for the total sample of 74 quarters. The most significant outliers are observed at the end of the sample and correspond to 2012Q4 and 2013Q1[12]. While using very tight significance level $\alpha = 0.01\%$, only 4 outliers were identified, but the mentioned above extreme observations remain. Importantly, that the final weight for irregular component, shown in the first graph in Figure 3, are generally consistent with the results of impulse indicator saturation at $\alpha = 1\%$. At the same time, the number of observations which have final weight for irregular component equal to 0 (the extreme values), almost coincides with the results of impulse indicator saturation technique at tight significance level.

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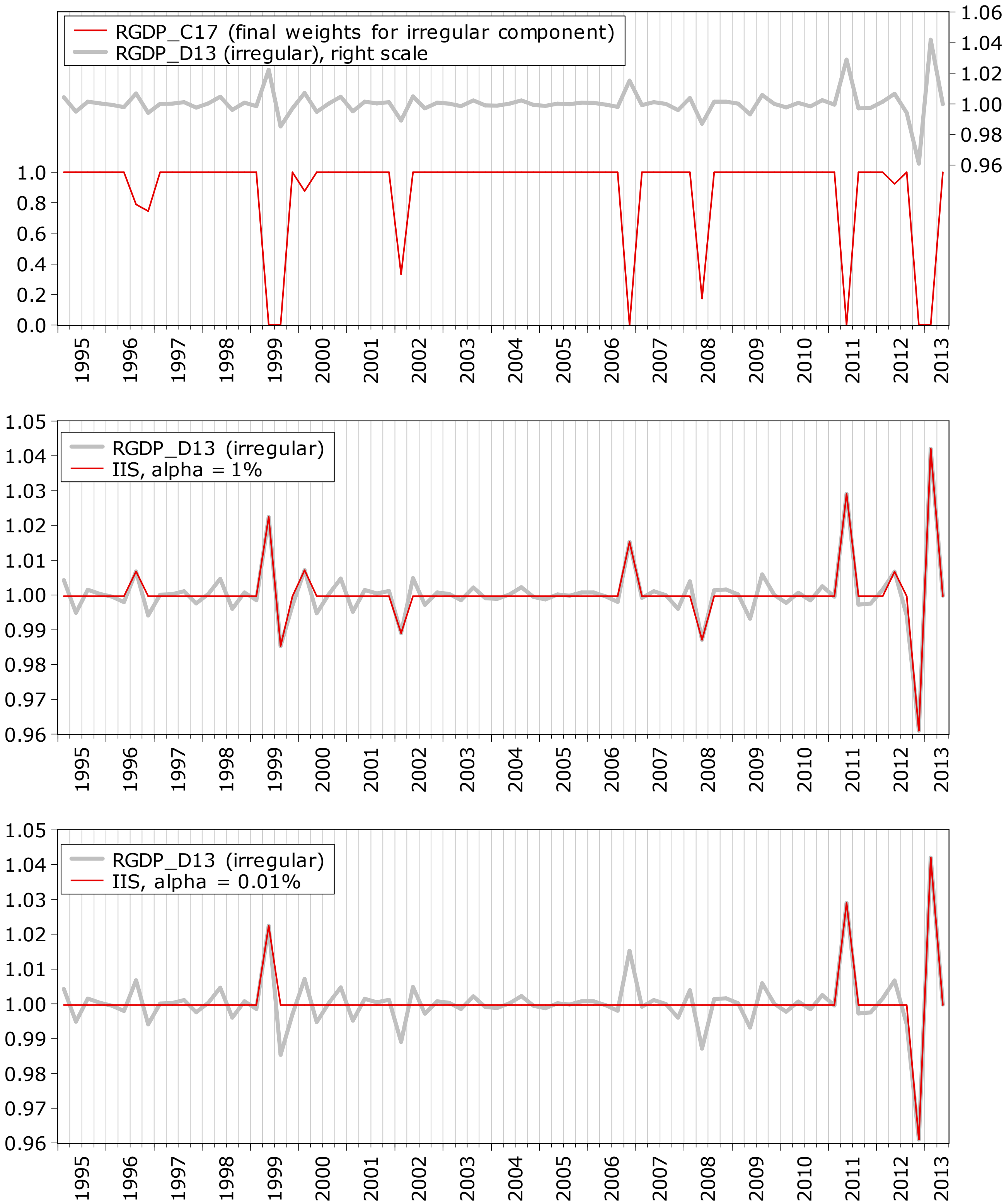
[11] Here and hereafter the letters D and S denote that components of time series decomposition are obtained by using X-12ARIMA and TRAMO/SEATS, respectively. We use conventional notations for components, used in standard software for seasonal adjustment.

[12] This is visually evident in the second and third graph of Figure 3. These graphs are illustrative representation of the regression (7) results which are not presented in the paper to reserve space, and can be available upon request. Let us remark here that the highest outliers have the largest coefficients in absolute value at the corresponding impulse indicator and the highest t-statistics.

Figure 3: Detecting data contaminations in real GDP



Note. Red line on the first graph depicts deviations of final weights for irregular from normal observations (weight = 1); any deviation from 1 means extreme value; the most sizable extreme values have the weight, which is equal to 0. Red lines on the second and third graphs represent the extreme values as the deviations from the mean, which is equal to 1. Alpha = 1% and 0.01% are significance levels for impulse indicator saturation (IIS) procedure.
Source: own estimations.

The obtained results show that the impulse indicator saturation allows to analyze quite successfully the behavior of irregular components and to identify extreme observation which may be due to data contamination. This method does not contradict with the results of table C17 from X-12ARIMA, and herewith can be used to solve a similar problem while using TRAMO/SEATS. Additionally, impulse indicator saturation technique confirms our working hypothesis about data contamination in 2012Q4 and 2013Q1, distorting the historical seasonal pattern.

Since the extreme observations, that are far away from the current period, has no significant effect on the results of seasonal adjustment, only the most recent outliers should be taken into account. In our case, there are the values for 2012Q4 and 2013Q1, and possibly for 2011Q2. Taken into account these considerations, we can replace the extreme values of the irregular component of the regression with the constant μ from (7), which is almost equal to 1 ($I^{\mu}_{t=iis}$). Then, based on (5a), seasonally adjusted data for the contaminated observations $Y^{sa}_{t=iis}$ will be equal to the value of trend component for the appropriate observation:

$$Y^{sa}_{t=iis} = T_{t=iis} \cdot I^{\mu}_{t=iis} = T_{t=iis} \,, \tag{8}$$

where $t = iis$ is observation in time series, identified as extreme by impulse indicator saturation techniques.

It should be noted, that in the future (after 2-3 years) the impact on seasonal adjustment procedures of contaminated observations in 2012Q4 and 2013Q1 will be reduced, but in 2013 this issue will remain in every quarter of the year.

4. Choosing the model for seasonal adjustment of Belarusian real GDP

4.1. X-12ARIMA or TRAMO/SEATS?

The previous section presented the results of the seasonal adjustment using seasonal filter X-11. It was shown that without identifying and taking into account the contaminated observations such an approach leads to wrong results. Adding the ARIMA(p, d, q)(P, D, Q) to seasonal adjustment may improve the results. As previously mentioned, there are the two competing approaches, namely X-12ARIMA and TRAMO/SEATS, which can be compared analysis using software X-13ARIMA-SEATS.

At first, we ran X-12ARIMA and TRAMO/SEATS in automatic mode. We used the standard specification RSA4c for the first method and RSA4 for the second. These specifications provide the automatic testing of the model for logarithmic transformation of the data (the choice of the multiplicative form of the decomposition of the time series), working day and Easter effects the automatic identification of outliers (additive, level shifts and temporary changes) and the automatic selection of the ARIMA(p, d, q)(P, D, Q). Then the resulting models are tested by standard set of tests[13]. Automatic selection in X-13ARIMA-SEATS is realized using the procedure similar to the method TRAMO, and finally leads to the following specification of ARIMA model: (1, 0, 0)(0, 1, 0). In general, such specifications for X-12ARIMA and TRAMO/SEATS have a satisfactory diagnosis. However, in both cases, the residuals of models are not normally distributed (the hypothesis of normality is rejected at 1% significance level due to a significant asymmetry in residuals distribution). We believe that this anomaly of the residuals is quite serious problem and a more appropriate model specification of ARIMA model is required.

Resulting in an automatic mode, ARIMA specification suggests that seasonal differences provide a stationarity of time series. The results of the unit root tests, however, have shown that the real GDP data apparently become stationary only after using of seasonal and regular differences. Therefore, on the next step we have fixed the values of d and D, assumed that the time series has a seasonal unit roots and regular unit root. Thereafter, an automatic selection procedure was retried. The result was a following specification of ARIMA model: (0, 1, 1)(0, 1, 0). Due to this specification, the problem of asymmetric distribution of the residuals was to a large extent resolve (the hypothesis of normality in this case is not rejected at the 5% significance level and the test for asymmetry of residuals is at marginal level close to the 5%).

[13] For more details see X-13ARIMA-SEATS (2013), Grudkowska (2012).

Since in our case X-12ARIMA and TRAMO/SEATS have the same specification of ARIMA model[14], appropriate diagnostic tests of both methods generally will be the same. The choice of the method for seasonal adjustment of real GDP was based on the following reasons.

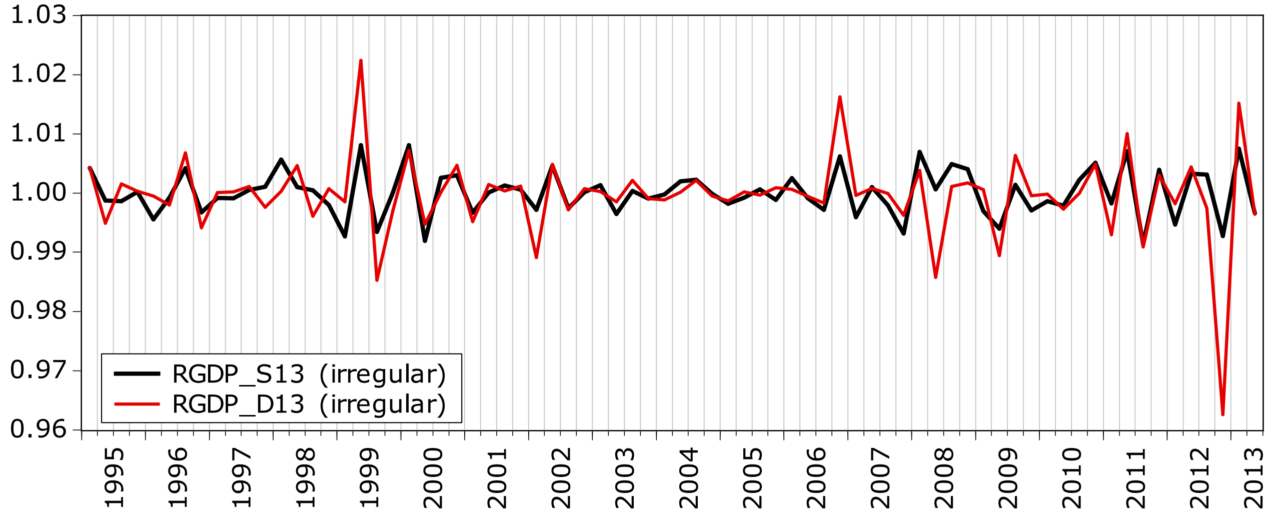
First, stability of seasonal adjustment is evaluated by analyzing revision history of seasonally adjusting data while adding new observations using the indicators characterizing the revision history of the seasonally adjusted data as you add new observations. The smaller the data revision, the more stable is the model used. Secondly, irregular components of methods are compared. The preference was given to the model, the irregular component of which has the least number of outliers requiring correction by impulse indicator saturation technique. Thirdly, the results of seasonal adjustment for the problematic quarters, especially for 2013Q1, are compared. Eventually, the model that provides more relevant results without correction for contaminated observations is preferred.

As for stability of the models, TRAMO/SEATS method is preferred. The values of the average absolute revisions of seasonally adjusted data and trend components are smaller for TRAMO/SEATS than for X-12ARIMA (see Annex C, Tables C1 and C2)[15].

Irregular components derived for the two approaches X-12ARIMA and TRAMO/SEATS are depicted in Figure 4. The volatility of irregular components for the first one is substantially higher than for the second approach. In the first case there are several very large outliers, while in the second case irregular components seem more homogenous. Impulse indicator saturation at 1% significance level identifies only 4 extreme observations for TRAMO/SEATS methods. At the same time, for X-12ARIMA methods 13 extreme observations are found. Consequently, an analysis of irregular components also favors TRAMO/SEATS method.

Utilization of X-12ARIMA provides seasonally adjusted quarter-to-quarter growth rate of real GDP in 2013Q1 equal to 1.060 or 6.0% and corresponding annualized becomes 1.263 or 26.3%. This is substantially less than with seasonal filter X-11, but still very far from real economic situation. TRAMO/SEATS method, even without correction for data contaminations gives more adequate results: 1.016 or 1.6% for quarter-to-quarter growth, and 1.063 or 6.3% for annualized growth rates. Thus, all mentioned above argues for TRAMO/SEATS as a most suitable method of seasonal adjustment of Belarusian real GDP.

Figure 4: Irregular components for X-12ARIMA and TRAMO/SEATS seasonal adjustment



Source: own estimations.

4.2. Seasonally adjusted Belarusian real GDP: Final results

In accordance with the results of previous analysis, for seasonal adjustment of Belarusian real GDP TRAMO/SEATS methods should be used with following ARIMA specification: (0, 1, 1)(0, 1, 0). This model has good diagnostics. It should be also noted, that in the process of automatic

[14] The specifications codes for X-13ARIMA-SEATS are given in Annex B, Table B1. Coupled with original data, that permits to replicate the results of seasonal adjustment, presented in this paper.
[15] The corresponding tests in Annex C, Tables C1 and C2 are marked in gray.

selection of the model outliers and other calendar effects not directly related with seasonal variation. Impulse indicator saturation identifies 4 extreme observations, one of which is related to 2013Q1. According to proposed approach, seasonally adjusted data for contaminated observations were replaced by appropriate trend component. On the basis of this data, year-to year and annualized seasonally adjusted growth rates of real GDP were calculated by using (3), (4) and (8).

Table 2: Seasonally adjusted growth rate of real GDP, % (1995Q1–2013Q2)

Year/quarter	TRAMO/SEATS, ARIMA (0, 1, 1)(0, 1 , 0) without corrections		TRAMO/SEATS, ARIMA (0, 1, 1)(0, 1, 0) with corrections	
	quarter to quarter	annualized	quarter to quarter	annualized
2009Q1	−1.17	−4.61	−1.17	−4.61
2009Q2	−0.43	−1.72	−0.43	−1.72
2009Q3	1.44	5.90	1.44	5.90
2009Q4	0.94	3.83	0.94	3.83
2010Q1	2.19	9.05	2.19	9.05
2010Q2	2.51	10.42	2.51	10.42
2010Q3	3.19	13.40	3.19	13.40
2010Q4	2.56	10.62	2.56	10.62
2011Q1	0.78	3.18	0.78	3.18
2011Q2	1.63	6.67	1.63	6.67
2011Q3	**−1.27**	**−4.99**	**−0.41**	**−1.65**
2011Q4	1.59	6.51	1.59	6.51
2012Q1	−0.48	−1.90	−0.48	−1.90
2012Q2	1.35	5.51	1.35	5.51
2012Q3	0.21	0.84	0.21	0.84
2012Q4	−0.99	−3.92	−0.99	−3.92
2013Q1	**1.55**	**6.34**	**0.79**	**3.21**
2013Q2	**−1.25**	**−4.89**	**−0.51**	**−2.01**

Source: own estimations.

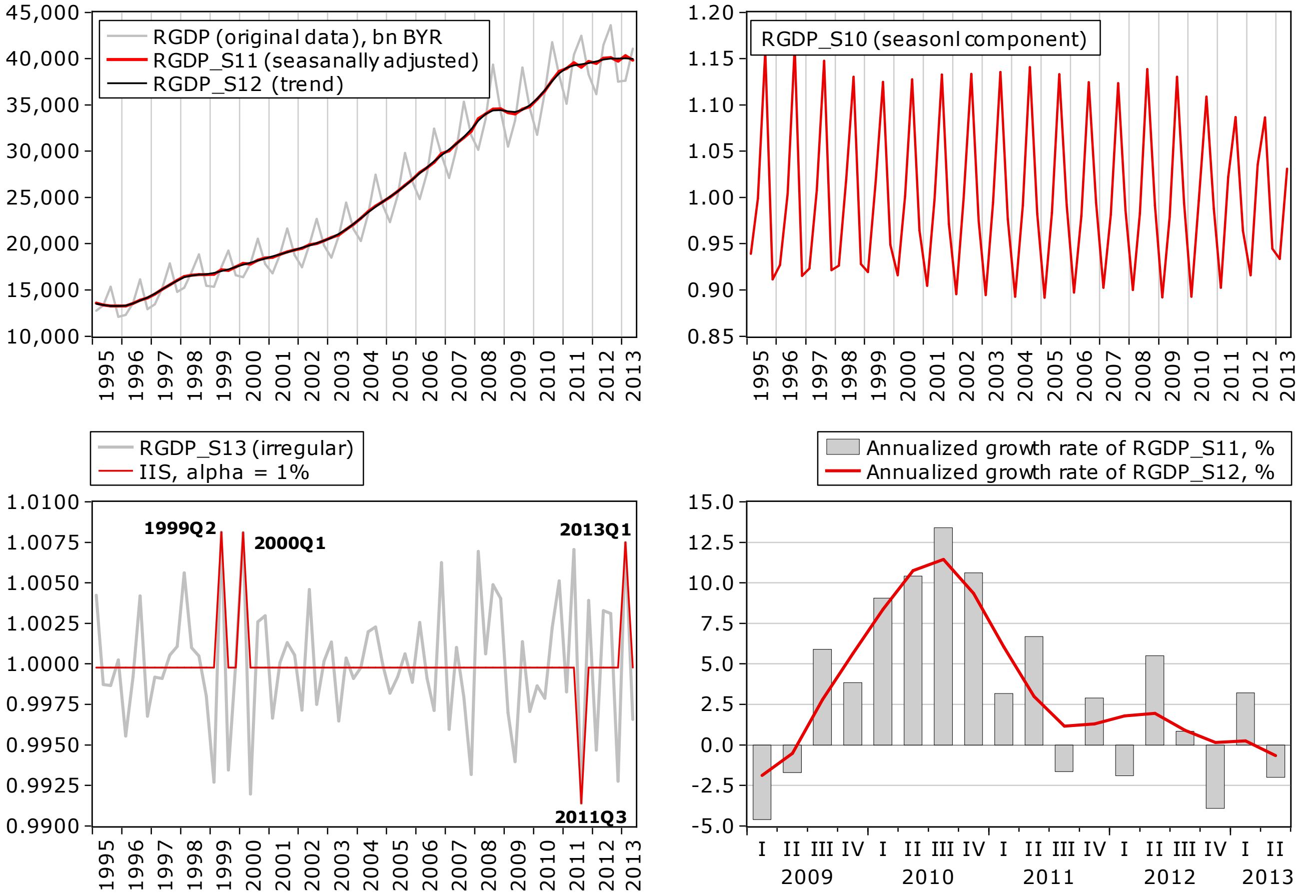
Table 2 presents the results of seasonal adjustment of Belarusian real GDP for the period 1995Q1-2013Q2. We provide for comparison the seasonally adjusted growth rates of real GDP with the corrections for data contaminations and without them. The results that differ are marked in bold.

The resulting growth rate for 2013Q1 differs substantially from those that were presented in the publications of the NBB and the IMF. Thus, without accounting for data contaminations, growth rate of real GDP in 2013Q1 is 1.55% and 6.44% for quarter-to-quarter and annualized growth rate, respectively. The result are considerably changed when corrections for data contamination in 2013Q1 is undertaken. In such a case a quarter-to quarter growth rate is equal to 0.79% for quarter-to quarter, and 3.21% for annualized value. The growth rate of real GDP in 2013Q2, according to our estimates, is -0.51% and -2.01% for quarter-to quarter and annualized values, respectively[16]. In our view, these results quite correctly reflected current economic situation, and the proposed approach permits to effectively resolve the problem of seasonal adjustment of Belarusian real GDP under data contaminations.

Figure 5 gives a visual representation of the results of the seasonal adjustment of real GDP. In the first graph, along with the original data, seasonally adjusted series for real GDP and its trend components without the corrections for data contaminations are shown. The second one depicts the dynamics of the seasonal component. The third graph represents the results of impulse indicator saturation technique implementation for identification of contaminated data. As can be seen, the usage of this method at 1% significance level allowed identifying four extreme observations and to make necessary adjustments. The fourth graph shows annualized growth rates of real GDP adjusted for extreme values and it corresponds with the results of data Table 2. Additionally, this graph represents appropriate growth rate of trend component, which are in line with the seasonally adjusted growth rate of real GDP.

[16] In accordance with our estimates, seasonally adjusted annualize growth rate of real GDP for the first half of 2013 is 0.56%.

Figure 5: Seasonal adjustment of real GDP with TRAMO/SEATS and corrections for data contaminations



Source: own estimations.

4.3. Does the analyzed time span really matter?

Until now we have used the data for the period 1995Q1-2013Q2 when conducting seasonal adjustment of Belarusian real GDP. The maximal long time span is of great importance for economic modeling and forecasting. However, the choice of the maximal length of the time series is not always being justified in terms of the quality of seasonal adjustment. If there are substantial structural breaks in the dynamics of the indicator, alterations of the seasonal patterns or significant changes in the statistical methodology, it may be appropriate to utilize a shorter time series to get better results of seasonal adjustment.

To check the stability of seasonal adjustment of real GDP in terms of time span, we applied TRAMO/SEATS procedure to a shorter time series, starting from 2002Q1. On the one hand, such a choice is arbitrary, but on the other hand, it provides sufficient number of observations for statistical methods of seasonal adjustment utilizations. Automatic model selection leads to the same specification of ARIMA model, namely (0, 1, 1)(0, 1, 0). Generally, the model has good diagnostic characteristics (see Annex C, Table C3). Impulse indicator saturation technique identifies two extreme observations in irregular component at 2.5% significance level (2011Q and 2013Q1).

The results are presented in Table 3. A comparison of the obtained results with those from Table 2 shows, that the truncation of the sample has almost no effect on the seasonally adjusted growth rate of real GDP. Thus, in our case, the length of the analyzed time series is not essential for the results of seasonal adjustment. This conclusion suggests the possibility of using sufficiently long time series for seasonal adjustment of real GDP. This is important for the analysis of long-term trends in real GDP, and econometric modeling and forecasting.

Table 3: Seasonally adjusted growth rate of real GDP, % (limited sample 2002Q1–2013Q2)

Year/quarter	ARIMA (0, 1, 1)(0, 1 , 0) without corrections		ARIMA (0, 1, 1)(0, 1, 0) with corrections	
	quarter to quarter	annualized	quarter to quarter	annualized
2009Q1	−1.18	−4.65	−1.18	−4.65
2009Q2	−0.44	−1.73	−0.44	−1.73
2009Q3	1.44	5.91	1.44	5.91
2009Q4	0.95	3.84	0.95	3.84
2010Q1	2.19	9.04	2.19	9.04
2010Q2	2.51	10.44	2.51	10.44
2010Q3	3.20	13.42	3.20	13.42
2010Q4	2.56	10.63	2.56	10.63
2011Q1	0.79	3.18	0.79	3.18
2011Q2	1.62	6.64	1.62	6.64
2011Q3	**−1.27**	**−5.00**	**−0.41**	**−1.64**
2011Q4	1.58	6.49	1.58	6.49
2012Q1	−0.47	−1.88	−0.47	−1.88
2012Q2	1.32	5.53	1.32	5.53
2012Q3	0.20	0.80	0.20	0.80
2012Q4	−0.99	−3.90	−0.99	−3.90
2013Q1	**1.55**	**6.36**	**0.80**	**3.23**
2013Q2	**−1.24**	**−4.86**	**−0.50**	**−1.97**

Source: own estimations.

5. Conclusions and policy recommendations

Seasonal adjustment is an important statistical procedure making it possible to remove seasonal variations from the data and show the real dynamics of economic indicators. The quality of the seasonal adjustment determines the final results and conclusions about the economic situation, needed for decision making.

The paper proposes an approach that allows conducting seasonal adjustment when several observations in analyzed time series are subject to data contamination and distort the final results. A dramatic example of such a situation is seasonally adjusted data for Belarusian real GDP in 2013Q1, when an application of standard statistical methods resulted in an annualized growth rate of about 40%.

The proposed approach has allowed identifying the contaminated observations and to carry out seasonal adjustment of real GDP eliminating an influence of these observations. As a result, according to our estimates, the corrected figures for Belarusian real GDP in 2013Q1 was 0.8% and 3.2% for quarter-to-quarter and annualized growth rates, respectively. This is fundamentally different from the results obtained initially by the NBB and the IMF. It should be noted, that the methodology and recommendations of this paper were taken into account by the NBB while preparing of the revised seasonally adjusted quarterly growth rate of real GDP for the first half of 2013 (NBB (2013a)). This approach can be used in any other situation when seasonal adjustment under data contaminations leads to the results which contradict to real economic situation.

This paper leads to a number of recommendations on seasonal adjustment of real GDP in Belarus:

(1) For seasonal adjustment of real GDP it is advisable to implement TRAMO/SEATS methods in automatic mode. This method allows us to get a model with good diagnostics. Additionally, the dynamic characteristics of the data and its order of integration should be taken into account. Appropriate econometric test have to be used for this purpose at the preliminary stage of analysis.

(2) Short-term alteration of seasonal pattern in the real GDP dynamics without explicit economic explanations can give evidence about possible data contaminations. To identify the extreme observations and possible data contaminations, we suggest investigating the behavior of an irregular component from decomposition of time series, and applied impulse indicator saturation techniques for this purpose. This methods permit to detect abnormal observation quite accurately. Impulse indicator saturation technique is implemented in econometric package OxMetrics.

(3) When calculating seasonally adjusted growth rate of real GDP under data contaminations, identified extreme observations should be replaced by appropriate values of trend component taken from decomposition of time series. This allows getting adequate estimates of economic growth.

(4) Time span for seasonal adjustment of real GDP actually does not influence the final results. This fact permits to obtain long enough time series that is of great importance for econometric modeling and forecasting.

(5) It should be taken into account that the usage of annualized growth rate under the data contaminations can increase the error of estimation manifold. If one faces a situation similar to those of 2013Q1, it is advisable to make correction proposed in this paper or temporary suspend the publication of seasonally adjusted data (supposedly within one year).

(6) In addition to the publication of seasonally adjusted growth rate of real GDP it is desirable to present corresponding data about trend component, which reflects the mid- and long-run tendencies of economic dynamics.

(7) It is advisable that Belstat carries out regularly seasonal adjustment of the main economic indicators for long enough time periods. Herewith, it is necessary to present the methodology and computations details. This will contribute to professional discussions on seasonal adjustment and improve the quality of seasonally adjusted economic data.

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Annex A. Raw data

Table A1. Belarusian real GDP in 2009 prices, billions of BYR

Year	Quarter			
	Q1	Q2	Q3	Q4
1995	12762.25	13365.20	15369.95	12096.60
1996	12317.77	13596.21	16165.89	12939.23
1997	13461.34	15162.01	17864.34	14782.54
1998	15260.05	16921.56	18847.76	15487.84
1999	15366.76	17454.35	19263.59	16597.79
2000	16387.79	17835.02	20555.54	17816.79
2001	16784.24	18799.52	21647.56	18776.55
2002	17468.96	19889.64	22667.81	19816.04
2003	18499.75	20854.99	24456.73	21579.78
2004	20311.08	23258.06	27449.57	24127.57
2005	22329.71	25227.49	29819.61	26721.13
2006	24845.03	27659.51	32403.15	29583.59
2007	27127.81	30270.11	35348.43	31691.81
2008	30170.32	33450.26	39353.92	34272.05
2009	30489.37	33302.60	39020.85	34629.35
2010	31785.76	36373.42	41761.09	38161.01
2011	35139.74	40441.99	42451.29	38257.46
2012	36179.70	41433.90	43576.30	37511.40
2013	37642.80	41057.39		

Source: own estimates based on Belstat data.

Annex B. Models specification

Table B1. X-13ARIMA-SEATS specification codes for final seasonal adjustment of real GDP

Automatic

```
series{
    file = "rgdp.txt"
    period = 4
    format = Datevalue
}
spectrum{
    savelog = peaks
}
transform{
    function = auto
}
regression{
    variables = ()
    aictest = ( td easter )
    savelog = aictest
}
outlier{
    types = ( AO LS TC )
}
automdl{
    savelog = amd
}
forecast{
    maxlead = 4
    print = none
}
estimate{
    print = (roots regcmatrix acm)
    savelog = (aicc aic bic hq afc)
}
check{
    print = all
    savelog = (lbq nrm)
}
seats{ }
slidingspans{
```

Automatic with fixed d=1 and D=1

```
series{
    file = "rgdp.txt"
    period = 4
    format = Datevalue
}
spectrum{
    savelog = peaks
}
transform{
    function = auto
}
regression{
    variables = ()
    aictest = ( td easter )
    savelog = aictest
}
outlier{
    types = ( AO LS TC )
}
 automdl{
    diff = (1,1)
    maxorder = (2,1)
    mixed = yes
    fcstlim = 15
    ljungboxlimit = 0.95
    checkmu = yes
    acceptdefault = no
savelog = amd
}
forecast{
    maxlead = 4
    print = none
}
estimate{
    print = (roots regcmatrix acm)
    savelog = (aicc aic bic hq afc)
```

`` savelog = percent additivesa = percent } history{ estimates = (fcst aic sadj sadjchng trend trendchng) savelog = (asa ach atr atc) } ```	``` } check{ print = all savelog = (lbq nrm) } seats{ } slidingspans{ savelog = percent additivesa = percent } history{ estimates = (fcst aic sadj sadjchng trend trendchng) savelog = (asa ach atr atc) } ```

Annex C. Models diagnostic

Table C1. X-12ARIMA diagnostic

```
Q-AUTO   rgdp   -------- -------- X-13ARIMA-SEATS run of rgdp
Automatic model chosen : (0 1 1)(0 1 0)

AICtd : none

AICeaster : rejected

Average Absolute Percentage Error : within-sample forecasts
AAPE(Last year)   :             2.40
AAPE(Last-1 year) :             9.15
AAPE(Last-2 year) :             1.65
AAPE(Last 3 years):             4.40
AIC  :    1101.4286
AICC :    1101.6104
BIC  :    1105.8968
Hannan-Quinn  :    1103.2013
No significant Ljung-Box Qs
Skewness coefficient:   -0.6005
Geary's a statistic:    0.7516
Kurtosis:    3.8311
Moving seasonality ratio     :       1.540
I/C Ratio                    :       0.243
Stable Seasonal F, B1 table  :     324.681
Stable Seasonal F, D8 table  :     365.760
Moving Seasonal F, D8 table  :       0.383
Identifiable seasonality     : yes
M01 :      0.024
M02 :      0.023
M03 :      0.000
M04 :      0.650
M05 :      0.200
M06 :      0.984
M07 :      0.106
M08 :      0.506
M09 :      0.280
M10 :      0.647
M11 :      0.604
Q   :      0.226
Q2  :      0.254

Percentage of quarters flagged as unstable.
Seasonal Factors :                                 0 out of  34 (  0.0 %)
Quarter-to-Quarter Changes in SA Series :          1 out of  33 (  3.0 %)
Year-to-Year Changes in SA Series :                0 out of  30 (  0.0 %)
AveAbsRev of Seasonal Adj. :                 1.149
AveAbsRev of Changes in Adj. :               1.389
AveAbsRev of Trend :                         1.261
AveAbsRev of Changes in Trend :              0.974
```

Table C2. TRAMO/SEATS diagnostic (total sample 1995Q1–2013Q2)

```
Q-AUTO   seats_ -------- -------- X-13ARIMA-SEATS run of seats_fix
Automatic model chosen : (0 1 1)(0 1 0)

AICtd : none

AICeaster : rejected

Average Absolute Percentage Error : within-sample forecasts
AAPE(Last year)   :             2.40
AAPE(Last-1 year) :             9.15
AAPE(Last-2 year) :             1.65
AAPE(Last 3 years):             4.40
AIC  :    1101.4286
AICC :    1101.6104
BIC  :    1105.8968
Hannan-Quinn  :    1103.2013
No significant Ljung-Box Qs

Skewness coefficient:   -0.6005
Geary's a statistic:    0.7516
Kurtosis:    3.8311

Percentage of quarters flagged as unstable.
Seasonal Factors :                              0 out of  22 (  0.0 %)
Quarter-to-Quarter Changes in SA Series :       0 out of  21 (  0.0 %)
Year-to-Year Changes in SA Series :             0 out of  18 (  0.0 %)
AveAbsRev of Seasonal Adj. :              0.993
AveAbsRev of Changes in Adj. :            0.930
AveAbsRev of Trend :                      0.955
AveAbsRev of Changes in Trend :           0.663
```

Table C3. TRAMO/SEATS diagnostic (reduced sample 2002Q1–2013Q2)

```
Q-AUTO   rgdp   -------- -------- X-13ARIMA-SEATS run of rgdp
Automatic model chosen : (0 1 1)(0 1 0)

AICtd : none

AICeaster : rejected

Average Absolute Percentage Error : within-sample forecasts
AAPE(Last year)   :             2.40
AAPE(Last-1 year) :             9.14
AAPE(Last-2 year) :             1.68
AAPE(Last 3 years):             4.41
AIC  :     676.5646
AICC :     676.8804
BIC  :     679.9917
Hannan-Quinn  :     677.8126
No significant Ljung-Box Qs
Skewness coefficient:   -0.7645
Geary's a statistic:    0.7296

Percentage of quarters flagged as unstable.
Seasonal Factors :                              0 out of  22 (  0.0 %)
Quarter-to-Quarter Changes in SA Series :       0 out of  21 (  0.0 %)
Year-to-Year Changes in SA Series :             0 out of  18 (  0.0 %)
AveAbsRev of Seasonal Adj. :              0.680
AveAbsRev of Changes in Adj. :            0.799
AveAbsRev of Trend :                      0.794
AveAbsRev of Changes in Trend :           0.465
```

Annex D. X-11 output: Outliers and corrections

Table D1. Final weights for irregular component (C17)

	1st	2nd	3rd	4th	S.D.
1995	100.0	100.0	100.0	100.0	0.6
1996	100.0	100.0	78.8	74.5	0.6
1997	100.0	100.0	100.0	100.0	0.6
1998	100.0	100.0	100.0	100.0	0.7
1999	100.0	0.0	0.0	100.0	0.6
2000	87.6	100.0	100.0	100.0	0.6
2001	100.0	100.0	100.0	100.0	0.6
2002	33.2	100.0	100.0	100.0	0.5
2003	100.0	100.0	100.0	100.0	0.4
2004	100.0	100.0	100.0	100.0	0.4
2005	100.0	100.0	100.0	100.0	0.2
2006	100.0	100.0	100.0	0.0	0.4
2007	100.0	100.0	100.0	100.0	0.6
2008	100.0	17.3	100.0	100.0	0.6
2009	100.0	100.0	100.0	100.0	0.9
2010	100.0	100.0	100.0	100.0	1.0
2011	100.0	0.0	100.0	100.0	1.0
2012	100.0	92.4	100.0	0.0	1.0
2013	0.0	100.0			1.0

Table D2. Modified seasonally adjusted series (D11 with D12 trend substituted whenever C17=0)

	1st	2nd	3rd	4th	TOTAL
1995	13775.	13356.	13260.	13235.	53627.
1996	13306.	13554.	13998.	14108.	54966.
1997	14558.	15070.	15571.	15998.	61197.
1998	16524.	16804.	16554.	16555.	66438.
1999	16694.	16996.	17257.	17501.	68448.
2000	17904.	17825.	18186.	18538.	72453.
2001	18493.	18841.	19150.	19361.	75844.
2002	19382.	19976.	20036.	20307.	79701.
2003	20619.	21008.	21585.	22034.	85246.
2004	22665.	23507.	24227.	24545.	94945.
2005	24909.	25570.	26347.	27114.	103940.
2006	27673.	28086.	28672.	29511.	113942.
2007	30184.	30778.	31284.	32089.	124334.
2008	33566.	33973.	34877.	34689.	137105.
2009	33967.	33642.	34729.	35098.	137436.
2010	35416.	36362.	37509.	38733.	148020.
2011	39093.	38876.	38470.	38923.	155361.
2012	40150.	40686.	39733.	39759.	160328.
2013	40018.	40215.			80233.