German Economic Team Belarus

IPM Research Center

Policy Paper Series [PP/02/2014]

Is the Output Gap a Useful Indicator for Monetary Policy in Belarus?

Igor Pelipas, Robert Kirchner, Enzo Weber

Berlin/Minsk, June 2014

About the IPM Research Center

The IPM Research Center was established in 1999 within the mutual project of the Institute for Privatization and Management (Minsk, Belarus) and CASE - Center for Social and Economic Research Foundation (Warsaw, Poland). It is a member of the CASE research network, William Davidson Institute NGO Alliance, and Economic Policy Institutes Network (project of the UNDP's Regional Bureau for Europe and the CIS). The IPM Research Center actively cooperates with the German Economic Team in Belarus (GET Belarus). Within this cooperation the IPM Research Center provides independent policy advice on economic issues to the different official agencies, namely to the Council of Ministers, National Bank, Ministry of Economy, Ministry of Finance and other organizations involved in the process of formation and implementation of economic policy.

The Mission of the IPM Research Center is to enhance national competitiveness through elaboration of the research-based economic policy recommendation and the promotion of professional dialogue on the urgent issues related to economic performance.

IPM Research Center

50B Zakharova Street, 220088, Minsk, Belarus

Tel:+375 (17) 2 100 105

Fax:+375 (17) 2 100 105

E-Mail: research@research.by

http://www.research.by

About the German Economic Team Belarus (GET Belarus)

The main purpose of GET Belarus is to conduct a dialogue on economic policy issues with the government, civil society, and international organizations. Experts of German Economic Team have experience in policy advice in several transition economies, including Ukraine, Russia, and Moldova. In Belarus the IPM Research Center and the German Economic Team provide information and analytical support to the Council of Ministers, the National Bank, the Ministry of Foreign Affairs, the Ministry of Economy and other institutions involved in the process of formation and implementation of economic policy.

German Economic Team Belarus

c/o Berlin Economics

Schillerstr. 59

D-10627 Berlin

Tel:+49 30 / 20 61 34 64 0

Fax: +49 30 / 20 61 34 64 9

E-Mail: info@get-belarus.de

http://www.get-belarus.de

Is the Output Gap a Useful Indicator for Monetary Policy in Belarus?

Executive Summary

The linkage between the real economy and inflation is traditionally of great importance for monetary authorities. In this context, the concept of the output gap plays a prominent role in conventional macroeconomic theory, applied research and monetary policy analysis. However, the concept must be operationalized in order to be used for monetary policy purposes. The main problem is that both potential output and the output gap as a derivative value from the former are not directly observable.

The central question of the paper is whether the output gap can be considered as a useful indicator for monetary policy in Belarus. We have tried to address this question from an empirical point of view, allowing the data to speak freely, and bearing in mind the considerations of economic theory. To this end, we considered two methods for estimating potential output and the output gap, where the dynamics of inflation is explicitly taken into account, namely structural vector autoregression (SVAR) and unobserved component (UC) model.

The output gap estimated on the basis of a SVAR model (without structural breaks in the variables taken into account) is very close to the output gap obtained by the National Bank of Belarus (2014), at least for the period 2009q1–2013q4. The output gaps derived from univariate and multivariate UC models, although remaining in negative territory at the end of the sample, have a clear-cut tendency to be closed. In accordance with conventional macroeconomic theory this means that **there is no reason for a softening of the stance of current monetary policy.**

A multivariate UC model, where real GDP, inflation and the output gap are estimated simultaneously, demonstrates a positive link between inflation and the output gap. The estimated coefficient is equal to 1.89. Univariate models of inflation confirm this finding and demonstrate that output gaps estimated on the basis of univariate and multivariate UC models are statistically significant and have a positive sign. The values of the coefficients of the output gap are varying, depending on the lag chosen in the model, from 0.74 to 2.03. The output gaps estimated on the basis of the UC models have definite information content in modelling inflation. UC models of inflation with the output gap outperform benchmark autoregressive models of inflation in the pseudo out-of-sample forecasts. These results are novel for the Belarusian economy and demonstrate the usefulness of unobserved component models for modelling the relationship between the output gap and inflation.

Authors

Igor Pelipas	pelipas@research.by +375 17 / 210 01 05
Robert Kirchner	kirchner@berlin-economics.com +49 30 / 20 61 34 64 2
Enzo Weber	enzo.weber@wiwi.uni-regensburg.de

Acknowledgements

The authors would like to express their gratitude to the staff of the Monetary Policy and Economic Analysis Directorate of the National Bank of the Republic of Belarus for providing helpful suggestions and comments. The usual disclaimer applies.

Contents

1. Introduction

The existing linkage between the real economy and inflation is traditionally of great importance for monetary authorities. In this context, the concept of the output gap plays a prominent role in conventional macroeconomic theory, applied research and monetary policy analysis. Theoretically, the output gap represents the difference between actual and potential output, where potential output is the level of output corresponding to the maximum output level that an economy can produce utilizing all available factors of production without causing inflationary pressures on economy. When actual output exceeds its potential, and the output gap becomes positive, this means there is demand pressure and an upturn in inflation. Such a situation indicates for monetary authorities that monetary policy needs to be tightened. On the contrary, a negative output gap, resulting from actual output below potential output, implies the need for monetary easing.

The concept of the output gap must be operationalized to be implemented for monetary policy. The main problem is that both potential output and the output gap as a derivative value from the former are not directly observable. In accordance with a vivid expression of Billmeier (2009), the output gap is a ghost, which must be caught by the use of statistical and econometric techniques. Thus, to be of practical usage, potential output and the output gap should be estimated. There are various approaches and methods for estimating potential output and the output gap proposed in the literature. It is well known, however, that all these estimates are characterized by substantial uncertainty. Thus, the choice of method of output gap estimation is a nontrivial task for monetary policy analysts.

In Belarus, different methods of potential output and output gap estimation were used. Important papers on this issue have been prepared at the National Bank of Republic of Belarus (NBB). For instance, Mironchik (2006) proposed to estimate the output gap using a calibrated small macroeconomic model and the multivariate Kalman filter. In Demidenko, Kuznetsov (2012) a production function approach, multivariate Hodrick-Prescott (HP) and Kalman filters were used in order to evaluate the potential output in Belarus and the main factors of its growth. It should be noted that in the first paper, the positive causal link from the output gap to inflation is predetermined in a calibrated model, in the second paper potential output and the output gap are not considered in the context of monetary policy at all; only equilibrium growth rates of the Belarusian economy are the subject of interest for the authors.

The output gap estimates coupled with other gaps are regularly presented in analytical reviews of the NBB, but without any quantitative connection to inflation (see, for example, NBB, 2014). Besides, the estimates of the output gap for the Belarusian economy traditionally appear in the country reports on Belarus by the International Monetary Fund (IMF). In accordance with these materials, the output gap is estimated by use of univariate filters such as the HP-filter and the Cristiano-Fitzgerald (CF) filter, as well as a structural vector autoregression (SVAR) approach (IMF, 2013; 2014). Any comments concerning the relationship of the output gap with inflation are absent.

It is true that studies where the authors try to relate the output gap and inflation and test econometrically such a relationship, are extremely scarce for Belarus. To the best of our knowledge, Kruk (2008) was among the few authors who tried to analyze the influence of the output gap on inflation. The main econometric finding from this paper can be summarizing as follows: In all specifications, the coefficients at the output gap with lags from 0 to 3 are either insignificant, or, if significant, have the wrong sign (i.e. an increasing output gap leads to a reduction of inflation). The author concludes that the overall predictive ability of the output gap concerning inflation is highly questionable in Belarus.

In this connection, a natural question arises: Why is the concept of the output gap then quite popular in the Belarusian context? What is the usefulness of this indicator, especially for monetary policy, if there is no empirical evidence for a meaningful association between the output gap and inflation in Belarusian data? In this paper, we have tried to address this question from an empirical point of view, allowing the data to speak freely, and bearing in mind the considerations of economic theory. To this end, we considered two methods for estimating potential output and the output gap, where the dynamics of inflation is taken into account in the relevant calculations. On this basis, we have evaluated the information content of the output gap as a variable affecting the dynamics of inflation, and determined its usefulness for monetary policy.

The rest of the paper is organized as follows. The second section briefly reviews the concept of the output gap and its main estimation approaches and methods. In the third section, the analytical framework of our estimations of the output gap is presented, namely two approaches are considered: The first one is based on the SVAR methodology, and the second approach is built around an unobserved component (UC) modelling, or so-called structural time series models. These approaches allow for the inclusion of inflation dynamics in the calculation of the output gap. The fourth section analyses the dynamic characteristics of the data used. In the fifth section the estimation results of the output gap using the methods mentioned above are presented and comparisons are made. Furthermore, the section is devoted to the econometric testing of the output gap as a predictor of inflation. The sixth section concludes and provides some policy implications.

2. Brief review of the concept and main estimation approaches

Potential output and the output gap traditionally play an important role in monetary policy, helping to predict and control inflation at low and stable levels. In general, potential output can be defined as the maximum level of output that an economy is able to produce *without generating an increase of inflation*[1]. In accordance with conventional macroeconomic theory potential output is determined by factors of production and technological level and represents the capacity of the national economy to supply goods and services to the consumers.

Within such a framework, the difference between actual output (say, real GDP) and potential output is considered as one of the main determinants of inflation pressure. This difference represents the output gap. In this line of reasoning, a positive output gap means that aggregate demand is higher than potential output, and inflation tends to increase. In turn, a negative output gap implies under-utilization of the economic potential and a decrease of inflation pressure. It follows that the output gap is an important indicator for modeling inflation dynamics and a useful tool for the conduct of monetary policy.

This elegant theoretical concept is faced with the practical necessity to get estimates of potential output and the output gap. The problem is that potential output, as well as the other variables characterizing the equilibrium states of economy, is directly unobservable and should be derived from observable macroeconomic data using appropriate techniques and models. However, the estimation of potential output is not an easy task. It can be obtained using a variety of approaches, ranging from purely statistical filters to the completely theoretically grounded dynamic stochastic general equilibrium (DSGE) models.

One can emphasize several approaches for estimating potential output and the output gap (see, for instance, Lim Choon Seng, 2007): Direct methods, univariate methods, multivariate and structural methods. This paper does not intend to consider the whole set of methods for potential output and output gap estimation, and refers the interested reader to the extensive literature on this issue. Instead, we focus on some of the most widely used methods of each of the mentioned groups.

The direct methods of potential output estimation are based on surveys of firms which provide useful insight on the degree of production capacities utilization of enterprises. However, it is difficult to operationalize the concept of potential output in questionnaires. Such data represents important first-hand information concerning the evaluation of capacity utilization, rather than a level of the production potential. Thus, direct methods are mostly adapted for the identification of business cycle turning points. The subjectivity of responses should also be taken into account.

The univariate statistical methods are widespread when estimating potential output and the output gap. These methods use only information contained in a single time series. The univariate HP-filter (Hodrick, Prescott, 1997) is apparently one of the most popular among them. The idea of HP-filter is to decompose a time series into unobservable components, namely trend and cycle, by using a two-sided moving average approach. A clear advantage of this method is

[1] See Hauptmeier et al. (2009) for details on history and discussion of the concept of potential output and output gap.

its simplicity and availability in all practically econometric packages. The trend obtained from the HP-filter implies potential output, while the cycle represents the output gap. Flexibility of estimation is assured by setting a special smoothing parameter λ (for quarterly data, this parameter is usually set equal to 1600). The main drawbacks of this method are also well-known: The arbitrariness of setting the smoothing parameter and the end-sample bias (the level of potential output is more affected by variations in actual output at the beginning and at the end of the sample), the possibility of spurious cyclicity when applying the HP-filter to integrated or near-integrated time series and the excessive smoothing of structural breaks.

There are other univariate methods that are used for potential output and output gap estimations: The univariate Beveridge-Nelson decomposition, Baxter-King and Cristiano-Fitzgerald filters, the univariate unobserved component model, etc. All these methods are available in various popular econometric packages. It is necessary to note one peculiarity of all these methods: They are purely statistical in nature, do not involve any theoretical considerations, and essentially produce a trend in the dynamics of real output. The output gap is eventually the deviation from this trend.

Multivariate and structural methods tend to introduce economic theory while estimating potential output and the output gap. Among them, one can mentioned the multivariate HP-filter, the multivariate unobserved component model, SVAR, production function approach and DSGE modeling. All multivariate methods permit to include additional variables which may be relevant from the point of view of economic theory. For a more detailed discussion on the various alternative methods of potential output estimation and its implication for economic policy analysis, see Cotis et al. (2004).

Additional words should be said about potential output estimation within various DSGE[2] models, which have become rather popular over the last decade. In Vetlov, et al. (2011) the notions of potential output within DSGE models are examined conceptually and empirically. Concerning the topic of our paper, the authors draw some interesting conclusions: (1) the comparison of DSGE model-based estimates of the output gap with traditional measures reveals that the two approaches may deliver significantly different estimates of the output gap. Like traditional estimates, DSGE model-based estimates of potential output are subject to high uncertainty that reflects real-time uncertainty, parameter uncertainty, as well as critical assumptions underlying the identification of the models' structural shocks; (2) there is no conclusive evidence proving that empirical estimates of model-consistent output gaps derived from larger and more realistic DSGE models are significantly better indicators of inflationary pressures than traditional measures; (3) the effects of the output gap on inflation and the size of the trade-off between output and inflation stabilization depends on the type of shocks and other structural features of the analyzed economy.

The existing variety of methods that can be used in estimations of potential output and the output gap leads to a substantial divergences of output gap estimates available for policy makers. A choice of the methods for better potential output and output gap estimation looks like a ghostbusting (Billmeier, 2004; 2009) in economic analysis. The unobservable nature of these variables makes it practically impossible to find the best measure, since the precise statistical errors of potential output estimates will never be known. In such a situation, different methods can lead to different policy recommendations. Which one, if any, should be chosen for practical usage?

In our view, the answer is straightforward: To be useful for monetary policy, output gap estimates should demonstrate statistically significant influence on inflation with the theoretically expected sign (a positive output gap should lead to an increase of inflation, while a negative output gap should lead to a decrease of inflation). Without such a requirement, the output gap becomes a worthless indicator for the conduct of monetary policy and can be used only as a measure of the discrepancy between actual and trend output in business cycle analysis without any relation to inflation. Of course, some reservations should be made. In case we must rely on poor-quality or distorted statistics, or observe a problematic conduct of economic policy, which breaks the natural links between macroeconomic indicators, an otherwise useful indicator might be rendered practically useless under such adverse conditions.

[2] See Morley (2010) for critical review of DSGE models.

It is important to note that inflation dynamics can be influenced by the various structural breaks, characterizing the different regimes of economic policy, internal and external shocks. These structural breaks can mask the real underlying relationship between the output gap and inflation. Therefore, such breaks should be taken into account while estimating the output gap and its link with inflation.

An essential requirement for choosing the method for potential output and output gap estimation is the availability of reliable data. Consequently, when macroeconomic data of interest are limited, a simpler method may be preferred, rather than more advanced methods with stronger theoretical grounds.

Thus, choosing from a variety of approaches and methods for potential output and output gap estimation, we rested on the following: (1) theoretical consideration should be taken into account explicitly in the model (at least, inflation dynamics should be incorporated into the model, so we need multivariate approaches); 2) data used for estimations should be immediately observable, available and reliable. Two of the above-mentioned methods meet the requirements, namely: SVAR and multivariate unobserved component model.

3. Analytical framework

In this paper we applied two methods of potential output and output gap estimation. The first one is based on a well-known and frequently cited paper by Blanchard, Quah (1989), where a Structural VAR (SVAR) model with long run identification restriction based on economic theory was used to estimate potential output and the output gap. The second one is built on the rather new paper by Harvey (2011), where the relationship between inflation and the output gap (Philips curve) is done with univariate and multivariate unobserved component models. The models used in our paper for potential output and output gap estimation are discussed below.

3.1. Structural VAR model

In Blanchard, Quah (1989) a macroeconomic model with only two variables (real GDP and the unemployment rate) is proposed, where real output is affected by two shocks: demand and supply. In accordance with the natural rate hypothesis, demand shocks have no long-run impact on the level of real GDP (demand shocks can have an effect on GDP in the short run only). On the contrary, supply-side or productivity shocks are supposed to have a permanent effect on output. The authors estimate a vector autoregression model with two variables and identify structural shocks, imposing the long-run restriction that demand shocks have only a temporary effect on real output.

Thus, the main idea of this bivariate SVAR model is to decompose real output into three components, namely (1) deterministic trend, (2) component determined by shocks, having a permanent effect on the supply side of the economy, (3) component determined by shocks that affect demand in the short run. The first two components represent potential GDP, while the latter can be considered as the output gap. It should be noted that within this model potential output and the output gap are determined simultaneously.

In our paper we adopted this method, using inflation instead of the unemployment rate. Thus, we use a bivariate SVAR model, included real GDP growth (seasonally adjusted), $\Delta rgdp_t^{sa}$ and inflation (seasonally adjusted), Δcpi_t^{sa}.[3] Thus, our system can be presented as vector of stationary covariance variables with expected value zero, $x_t = \left[\Delta rgdp_t^{sa},\ \Delta cpi_t^{sa}\right]'$. The SVAR model can be expressed as an infinite moving average representation of real GDP and inflation:

$$x_t = A(L)\varepsilon_t = \sum\nolimits_{i=0}^{\infty} A_i \varepsilon_{t-i} \ , \qquad (1)$$

where $A(L)$ is a 2×2 lag polynomial; $\varepsilon_t = \left[\varepsilon_t^s, \varepsilon_t^d\right]'$ is a vector of exogenous, unobserved structural shocks (supply and demand shock respectively), that satisfies $E\left[\varepsilon_t\right] = 0$ and $E\left[\varepsilon_t \varepsilon_t'\right] = I$.

[3] The nomenclature of the variables used is presented in Table 1.

In order to identify the structural model, one should estimate the following reduced-form VAR:

$$x_t = \Phi(L)x_t + e_t = \sum_{i=0}^{p} \Phi_i x_{i-t} + e_t, \quad (2)$$

where $\Phi(L)$ is a 2×2 lag polynomial of order *p*; e_t is a vector of estimated reduced-form residuals with $E[e_t] = 0$, and $E[e_t e_t'] = \Sigma$.

The reduced form can be inverted using the Wold decomposition, resulting in the reduced-form moving-average representation:

$$x_t = C(L)e_t = \sum_{i=0}^{\infty} C_i e_{t-i}, \quad (3)$$

where $C(L)$ is a lag polynomial that can be expressed in terms of $\Phi(L)$ as follows: $C(L) = [1 - \Phi(L)L]^{-1}$.

From (1) and (2) one can see that the reduced-form innovations e are linearly related to the structural innovations ε:

$$e_t = A_0 \varepsilon_t, \quad (4)$$

where A_0 is 2×2 matrix of the contemporaneius effects of structural innovations. Herewith

$$E[e_t e_t'] = A_0 E[\varepsilon_t \varepsilon_t'] A_0'. \quad (5)$$

Since $E[\varepsilon_t \varepsilon_t'] = I$, then

$$A_0 A_0' = \Sigma. \quad (6)$$

In order to recover the structural innovations, it is necessary to impose sufficient identification restrictions to identify the elements of matrix A_0. Three pieces of information are obtained from the symmetric 2×2 matrix $\Sigma = A_0 A_0'$. Thus, one identification restriction is needed to be imposed in order to recover the four unknown elements in A_0. This restriction is based on economic theory and supposes that demand shocks have no long-run effect on real GDP:[4]

$$\sum_{i=0}^{\infty} A_i(1,2) = 0 \quad . \quad (7)$$

where $A_i(i, j)$ represents the elements in row i and column j of matrix A_i.

The residuals from the unrestricted VAR and the estimated parameters of A_0 can be used to construct the vector of exogenous structural shocks. Since potential GDP corresponds to the permanent component of GDP in the system, the equation for growth of potential GDP can be obtained using the vector of supply shocks:

$$\Delta rgdp_t^{sa,\,potential} = \sum_{i=0}^{\infty} A_i(1,1)\varepsilon_t^s. \quad (8)$$

Similarly, growth of the output gap can be derived as follows:

$$\Delta rgdp_t^{sa,gap} = \sum_{i=0}^{\infty} A_i(1,2)\varepsilon_t^d \quad . \quad (9)$$

The levels of potential output and the output gap can be easily obtained.

For estimation of potential output and the output gap using the methodology described above, we utilize the Add-in procedure in Eviews econometric software (HDecomp) that provides a procedure that decomposes the historical values of time series from a VAR estimations[5].

[4] For the role of persistence when using long-run restrictions, see Tschernig et al. (2013, 2014).
[5] This Add-in procedure is provided by Eren Ocakverdi, see http://www.eviews.com/Addins/addins.shtml.

3.2. Unobserved component model

In Harvey (2011) the relationship between inflation and the output gap is modelled by an unobserved component model. Both univariate and multivariate models are discussed. In the first case, the output gap is obtained by using a univariate unobserved component model for output. Then the output gap is included in a univariate model of inflation. In a multivariate case, output, inflation and the output gap are modelled simultaneously.

In the univariate model, the output gap can be estimated from the unobserved component model for real output. A trend-cycle model can be expressed as follows:

$$rgdp_t^{sa} = \mu_t + \psi_t + \varepsilon_t, \quad t = 1, \dots, T\ , \tag{10}$$

where μ_t is an integrated random walk,

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, \\ \beta_t &= \beta_{t-1} + \xi_t, \qquad t = 1, \dots, T\ , \end{aligned} \tag{11}$$

ψ_t is a stochastic cycle, β_t – slope, $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$, $\eta_t \sim NID(0, \sigma_\eta^2)$, and $\xi_t \sim NID(0, \sigma_\xi^2)$.

The stochastic cycle

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \quad t = 1, \dots, T, \tag{12}$$

where λ_c is frequency in radians, ρ is a damping factor, with $0 \le \rho \le 1$, and κ_t, κ_t^* are two mutually independent white noise disturbances with zero mean and common variance σ_k^2. The disturbances $\varepsilon_t, \xi_t, \kappa_t, \kappa_t^*$ are serially and mutually uncorrelated with variances σ_ε^2 and σ_ξ^2 for irregular and slope.

In this model, the smoothed estimates of the cycle can be considered as the output gap. It should be noted that the HP-filter can be considered as a special case of this unobserved component model. However, the unobserved component model has no "end of the sample problem", and can be used for forecasting of potential output and the output gap.

Since trend inflation is well approximated by a driftless random walk, it can be modelled as a random walk plus noise or local level model:

$$\Delta cpi_t^{sa} = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \quad t = 1, \dots, T\ , \tag{13}$$

$$\mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2), \quad t = 1, \dots, T\ . \tag{14}$$

The disturbances ε_t and η_t are serially and mutually uncorrelated, normally and independently distributed with zero mean and variance σ^2.

In fact, μ_t in (11) represents core (trend) inflation. By analogy to the model for output, a stochastic cycle ψ_t can also be added to the model of inflation. To test a relationship between inflation and output gap in a univariate setting, a model of inflation should be expanded by inclusion of output gap with some lags:

$$\Delta cpi_t^{sa} = \mu_t + \psi_t + a_1 x_{t-j} + \varepsilon_t, \quad t = 1, \dots, T\ , \tag{15}$$

where x_{t-j} is the output gap from the model (11–12), and $j = 0, 1, 2$ in our case.[6]

In a multivariate setting inflation and output are modelled simultaneously as follows:

[6] Only one lag is used at a time in the equation (15).

$$\begin{bmatrix} \Delta cpi_t^{sa} \\ rgdp_t^{sa} \end{bmatrix} = \begin{bmatrix} \mu_t^{\Delta cpi} \\ \mu_t^{rgdp} \end{bmatrix} + \begin{bmatrix} \psi_t^{\Delta cpi} \\ \psi_t^{rgdp} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\Delta cpi} \\ \varepsilon_t^{rgdp} \end{bmatrix}, \tag{16}$$

where $\mu_t^{\Delta cpi}$ is a random walk as in (14) and μ_t^{rgdp} is an integrate random walk as in (11).

The stochastic cycles are modelled as 'similar cycles', so that if $\boldsymbol{\psi}_t = (\psi_t^{\Delta cpi}, \psi_t^{rgdp})'$ then

$$\begin{bmatrix} \boldsymbol{\psi}_t \\ \boldsymbol{\psi}_t^* \end{bmatrix} = \left[\rho \begin{pmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda \end{pmatrix} \otimes \mathbf{I}_2 \right] \begin{bmatrix} \boldsymbol{\psi}_{t-1} \\ \boldsymbol{\psi}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\kappa}_t \\ \boldsymbol{\kappa}_t^* \end{bmatrix}, \quad t = 1, \ldots, T, \tag{17}$$

where $\boldsymbol{\kappa}_t$ and $\boldsymbol{\kappa}_t^*$ are 2×1 vector of the disturbances such as $E(\boldsymbol{\kappa}_t \boldsymbol{\kappa}_t') = E(\boldsymbol{\kappa}_t^* \boldsymbol{\kappa}_t^{*\prime}) = \Sigma_k$, where Σ_k is a 2×2 covariance matrix, and $E(\boldsymbol{\kappa}_t \boldsymbol{\kappa}_t^{*\prime}) = 0$.

The cycle of inflation can be broken down into two independent parts, one of which depends on real GDP cycle, that is $\psi_t^{\Delta cpi} = \beta \psi_t^{rgdp} + \psi_t^{\Delta cpi\dagger}$, where

$$\beta = Cov(\psi_t^{\Delta cpi}, \psi_t^{rgdp}) / Var(\psi_t^{rgdp}) = Cov(\kappa_t^{\Delta cpi}, \kappa_t^{rgdp}) / Var(\kappa_t^{rgdp}), \tag{18}$$

and $\psi_t^{\Delta cpi\dagger}$ is cyclical component specific to inflation.

Substituting in the inflation equation in (16) gives

$$\Delta cpi_t^{sa} = \mu_t^{\Delta cpi} + \beta \psi_t^{rgdp} + \psi_t^{\Delta cpi\dagger} + \varepsilon_t^{\Delta cpi}. \tag{19}$$

If the cycle disturbances $\kappa_t^{\Delta cpi}$ and κ_t^{rgdp} are perfectly correlated, (19) corresponds to the relationship between inflation and the output gap with zero lag.

We estimated univariate and multivariate unobserved component models using module Stamp 8.3 of OxMetrics 7.0 software.[7] It should be noted that possible structural breaks in the variables have to be introduced into UC models to render their dynamics properly. In the next section we detect multiple structural breaks in the variables and use them in the following estimations. Besides, in STAMP 8.3 an automatic detection procedure for detection of outliers, level shifts and trend breaks is implemented. The program is able to propose a set of potential outliers, level shifts and trend breaks for univariate and multivariate time series on the basis of two-step procedure based on the auxiliary residuals. First the selected model is estimated and the diagnostics are investigated. Then a first set of potential outliers, shifts and breaks are selected from the auxiliary residuals. After re-estimation of the model, only those interventions survive that are sufficiently significant (see Koopman et al. (2009)). This procedure is also utilized in further analysis.

4. Data used and their dynamic characteristics

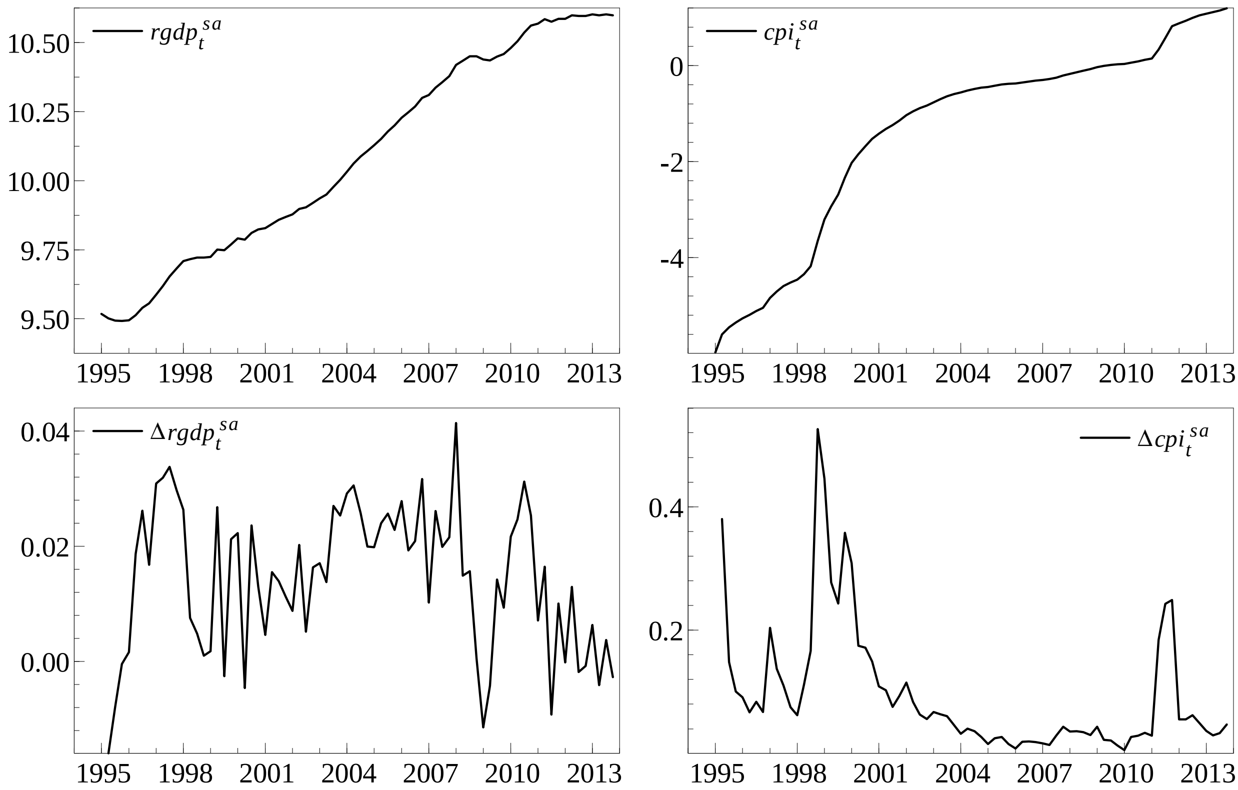
For econometric modelling, we used quarterly data of real GDP in average 2009 prices and CPI index over 1995q1–2013q4. Official statistics do not represent the real GDP data in average 2009 prices for the considered period. Thus, the real GDP data for a number of years in 1995, 2000 and 2005 prices were converted into real GDP in 2009 prices by using available quarterly growth rates of real GDP. Quarterly CPI index is obtained by averaging monthly data.

Then the raw data (see Annex, Table A1) were tested for seasonality and, if necessary, the appropriate adjustment was made. We used X-13ARIMA-SEATS procedure for seasonal adjustment.[8] To obtain seasonally adjusted data, it is necessary to specify correctly the ARIMA(p, d, q)(P, D, Q) model. Within X-13ARIMA-SEATS the choice of d and D can be done automatically. However, the order of integration of data determined in the automatic mode is sometimes

[7] See Koopman, Harvey, Doornik, Shephard (2009).
[8] For details see http://www.census.gov/srd/www/x13as.

not consistent with the actual dynamic characteristics of the time series. Therefore, to determine the order of integration of the variables, we tested them for seasonal unit root, using HEGY-test (Hylleberg, Engle, Granger, Yoo, 1990)[9]. Visual inspection of data and a formal econometric analysis suggests that the real GDP and CPI indices are non-stationary variables and contain regular and seasonal unit roots. Seasonal differences do not make the time series stationary. To ensure stationarity, one has to use also the first difference. Based on these results, we apply seasonal adjustment semi-automatic selection of ARIMA(p, d, q)(P, D, Q) with fixed parameters d=1 and D=1. Seasonally adjusted time series in natural logarithms are presented in Figure 1. The first difference of these time series approximate the growth rate of the real GDP and inflation.

Figure 1. Data used (log scale, seasonally adjusted)



Source: own estimations.

Conventional unit root tests give ambiguous and somewhat contradictory results for the variables in first differences. Specifically, for Δcpi_t^{sa} a standard (augmented) ADF-test clearly rejected the null hypothesis of unit root, while ADFGLS-test (Elliot, et al., 1996) did not reject the null. At the same time, KPSS-test (Kwiatkowski, et al., 1992) rejected the null hypothesis of stationarity at 5%. For $\Delta rgdp_t^{sa}$ unit root is rejected by ADF-test and stationarity is not rejected by KPSS-test whereas ADFGLS-test did not reject the null hypothesis of unit root. Visual inspection of the time series shows that multiple structural breaks (level shifts) in the first differences the analyzed data (Figure 1) may exist. Bearing in mind that conventional unit root test are strongly affected by such breaks, we first specify them in a formal setting.

To identify such structural breaks we employed the Bai-Perron multiple breakpoint test (Bai, Perron, 1998; 2003) for the first differences of the time series. Within this approach, the sum of squared residuals is minimized in order to identify the dates of k structural breaks in time series Δy_t and, thereby, determine $k+1$ regime in dynamics of the examined variable, on the basis of the following model: $\Delta y_t = \gamma_{k+1} + \tau_t$, where Δy_t is a variable of interest; γ_{k+1} is a series of $k+1$ constants, that characterized the means of the variable in each of $k+1$ regimes; τ_t

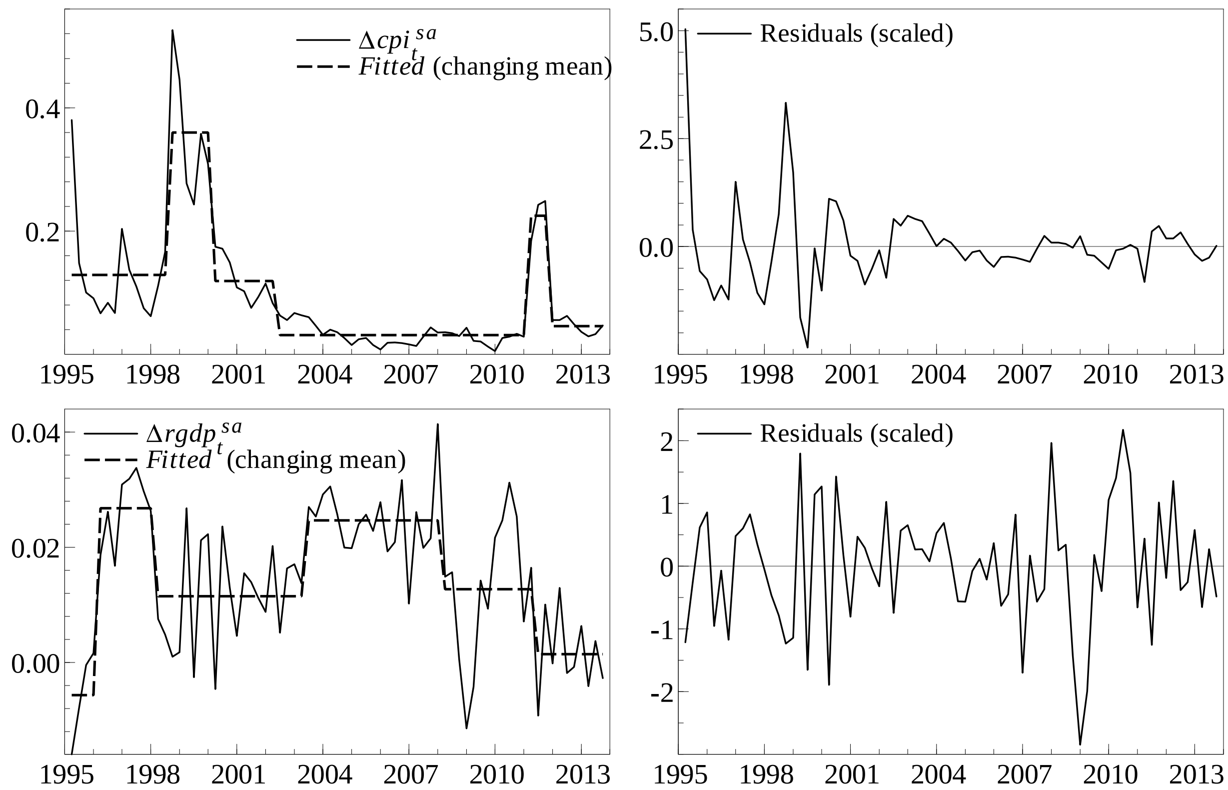
[9] See Annex, Table A1.

are regression residuals. This is corrected for autocorrelation by reservation of a certain share of the sample corresponding to minimal regime duration (0.05 from the total sample in our case). The final model is chosen using Bayesian information criterion (BIC).

Figure 2 depicts the results of these tests in graphical form[10]. As one can see, five break points were found for inflation (1998q4, 2000q2, 2002q3, 2011q2, 2012q1) and as many as for GDP growth (1996q2, 1998q2, 2003q3, 2008q2, 2011q3). When such breaks are taken into account, one can conclude that these variables are eventually stationary with changing means. Formal unit root tests with multiple structural breaks clearly reject the null hypothesis of unit root both for Δcpi_t^{sa} and $\Delta rgdp_t^{sa}$ (t-ADF is equal to −10.22 and – 7.52 accordingly, which exceeds the critical value at 1% significance level).[11]

This fact plays an important role in analyzing the relationship between inflation and the output gap, and appropriate step dummies (if significant) should be included in econometric models to reflect properly the dynamics of inflation.

Figure 2. Bai-Perron test for multiple structural breaks



Source: own estimations.

[10] Formal results of these tests are presented in the Annex, Table A2.

[11] In Pelipas (2011, 2012) it is argued that an augmented Dickey-Fuller unit root test is intrinsically a univariate case of the vector autoregression model with an equilibrium correction mechanism. If a variable of interest is stationary, then it is cointegrated with itself. This means that any departure of a variable from its equilibrium level after a shock will be corrected. In fact, this is similar to the feedback coefficients in Johansen's multivariate cointegration model that characterize the speed of the equilibrium correction in the system. In this context, it is possible to reformulate Dickey-Fuller unit root test, treating the multiple changes of the mean defined endogenously as in the vector autoregression model with equilibrium correction mechanism in the case when a constant are restricted in cointegration space. The appropriate coefficient in the model one can treat as an equilibrium correction mechanism and its significance can be tested using critical values from the cointegration test for conditional equilibrium correction model (see Ericsson and MacKinnon (2002)). The step dummies in a model can be considered as the additional variables in cointegration vector and then one can use the critical values in accordance with the total number of such variables. If the break points are determined, proposed approach permits unit root testing for any number of structural breaks.

Below we present a detailed description of the data and all transformations of the time series used in this study.

Table 1. Variables used

Nomenclature	Variable	Definition	Source
$RGDP_t$	Real gross domestic product (GDP) in average 2009 prices, quarterly data, billions of BYR	-	Own estimations based on Belstat
CPI_t	Consumer price index (CPI), quarterly data, 2009=1	Average from monthly data	Own estimations based on Belstat data
$RGDP_t^{sa}$	Seasonally adjusted real GDP	X-13ARIMA-SEATS: automatic TRAMO-SEATS procedure with fixed d=1, D=1 and final specification of ARIMA(0, 1, 1)(0, 1, 0); contaminated data for 2013q1 was corrected; clearly significant seasonality is identified	Own estimations
CPI_t^{sa}	Seasonally adjusted CPI	X-13ARIMA-SEATS: automatic TRAMO-SEATS procedure with fixed d=1, D=1 and final specification of ARIMA(1, 1, 1)(0, 1, 1); clearly significant seasonality is identified	Own estimations
$rgdp_t$	Natural logarithm (ln) of real GDP	$rgdp_t = \ln RGP_t$	Own estimations
cpi_t	Natural logarithm (ln) of CPI	$cpi_t = \ln CPI_t$	Own estimations
$\Delta rgdp_t$	Logarithmic first differences of real GDP	$\Delta rgdp_t = rgdp_t - rgdp_{t-1}$	Own estimations
Δcpi_t	Logarithmic first differences of CPI	$\Delta cpi_t = cpi_t - cpi_{t-1}$	Own estimations
$\Delta_4 rgdp_t$	Logarithmic fourth (seasonal) differences of real GDP	$\Delta_4 rgdp_t = rgdp_t - rgdp_{t-4}$	Own estimations
$\Delta_4 cpi_t$	Logarithmic fourth (seasonal) differences of CPI	$\Delta_4 cpi_t = cpi_t - cpi_{t-4}$	Own estimations
$rgdp_t^{sa}$	Natural logarithm (ln) of seasonally adjusted real GDP	$rgdp_t^{sa} = \ln RGP_t^{sa}$	Own estimations
cpi_t^{sa}	Natural logarithm (ln) of seasonally adjusted CPI	$cpi_t^{sa} = \ln CPI_t^{sa}$	Own estimations
$\Delta rgdp_t^{sa}$	Logarithmic first differences of seasonally adjusted real GDP	$\Delta rgdp_t^{sa} = rgdp_t^{sa} - rgdp_{t-1}^{sa}$	Own estimations
Δcpi_t^{sa}	Logarithmic first differences of seasonally adjusted CPI	$\Delta cpi_t^{sa} = cpi_t^{sa} - cpi_{t-1}^{sa}$	Own estimations
GAP_{t-j}	Different measures of output gap, in logs	Estimated by using HP-filter, SVAR model, UC model	Own estimations

5. Estimation results

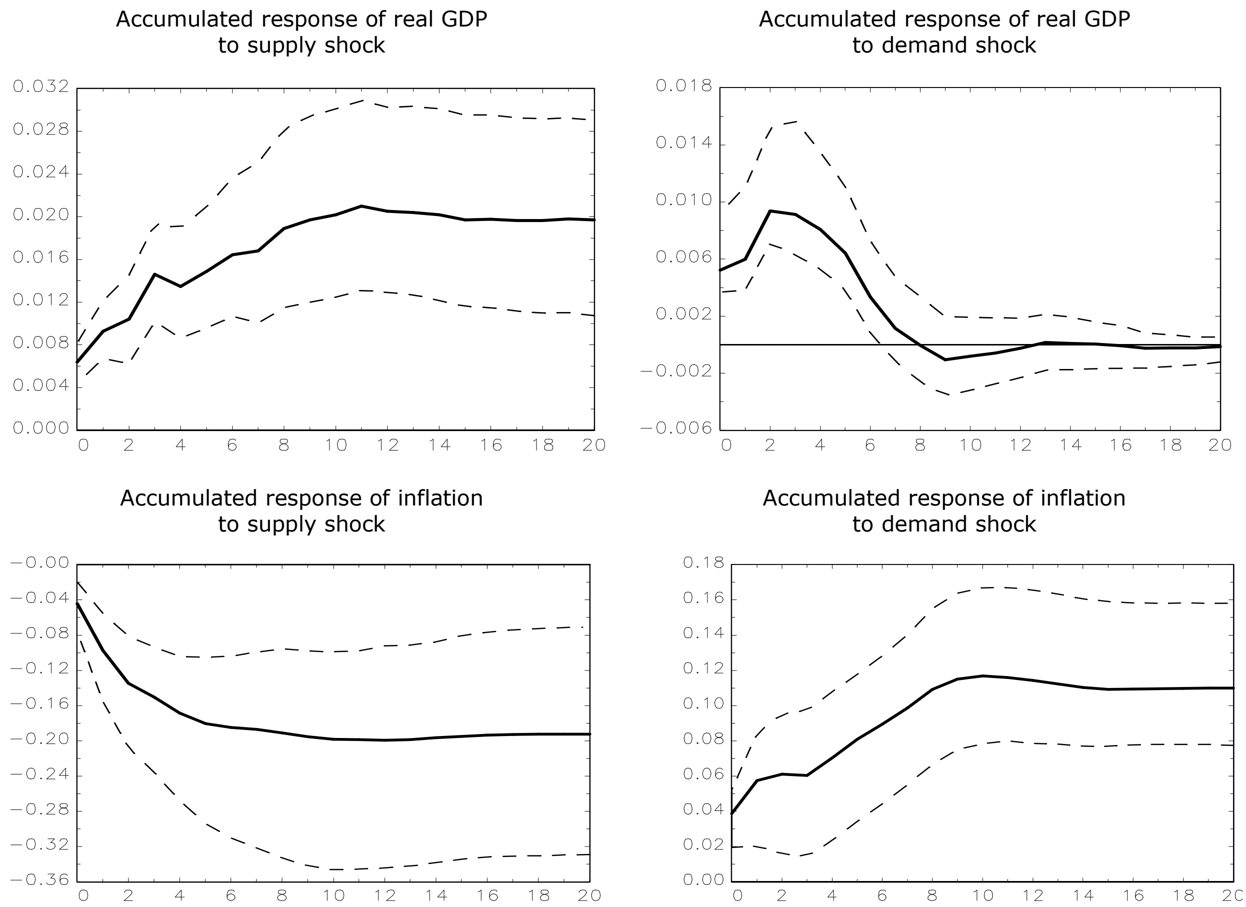
5.1. Structural vector autoregression model

In order to estimate the output gap within a SVAR model, we specify a VAR with 5 lags, constant and trend. This specification is consistent with the data. The order of the VAR is chosen on the basis of LR-tests (sequential modified LR test statistic), and a variety of criterions, namely FPE (final prediction error), AIC (Akaike information criterion) and HQ (Hannan-Quinn information criterion). All of them suggest 5 lags as an optimal order of the VAR (only Schwarz information criterion suggests 1 lag in VAR that does not properly reflect the dynamics of the data). In general, a VAR with 5 lags passes the specification test. The null hypothesis of no serial correlation up to 5 lags and residual heteroskedasticity are not rejected at any convenient levels. There is evidence of non-normality in residuals in the equation for Δcpi_t^{sa}. The equation for $\Delta rgdp_t^{sa}$ passed the residual normality test. Deterministic terms (constant and trend) are significant in the chosen specification.

Then we imposed the long-run restriction in accordance with (7) and estimate the SVAR model. Impulse response functions due to supply and demand shocks are depicted in Figure 3. The dynamic effects of supply and demand shocks are in line with the theoretical considerations. A supply shock leads to an increase in real GDP in the long run that stabilizes after 10 quarters. In turn, inflation at first decreases after a supply shock and then stabilizes after 10 quarters, in fact mirroring the response of real GPD with opposite sign. The impulse responses characterizing the effects of a supply shock on output and inflation are statistically significant according to 95% bootstrap confidence intervals.

A demand shock has a short-run effect on real GDP. It leads to an increase of output up to 2 quarter and then this effect tends to approach zero after approximately 6 quarters (then the impulse response function becomes insignificant). Inflation grows after a demand shock and stabilizes after 10–12 quarters. The impulse responses characterizing the effects of a demand shock on inflation are statistically significant at the 95% significance level.

Figure 3. Impulse response functions

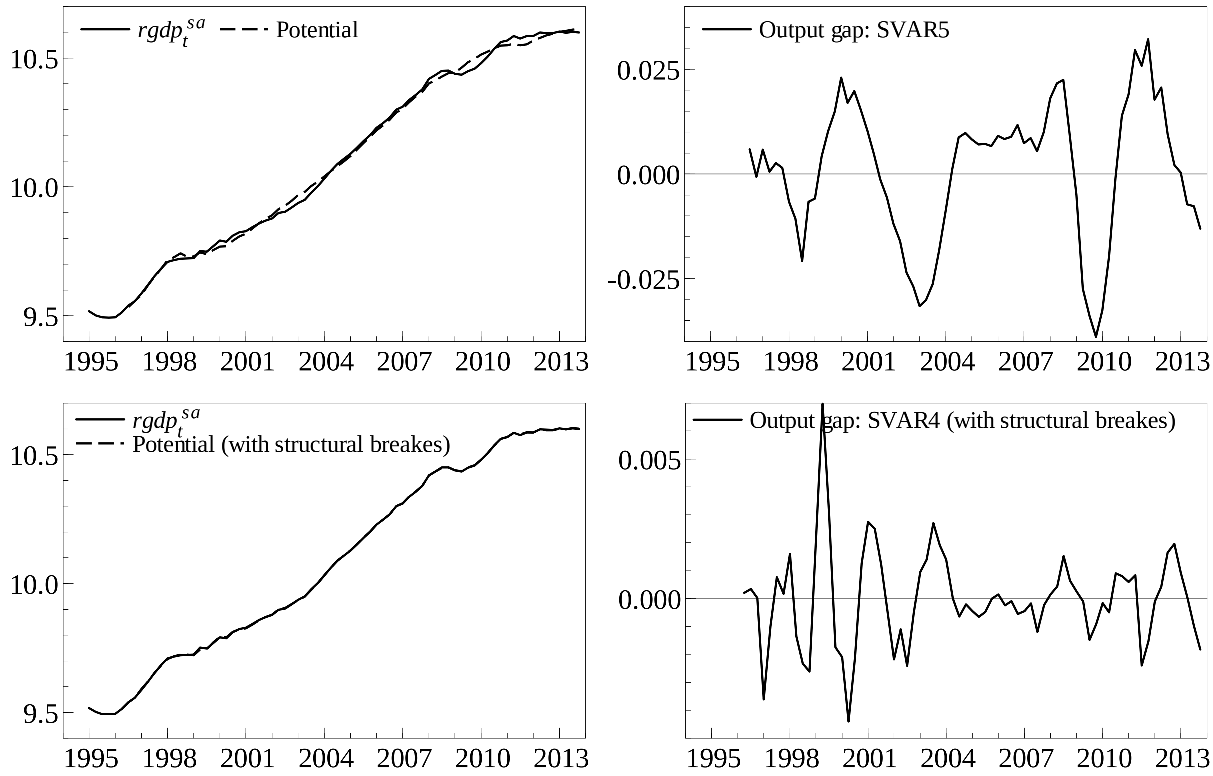


Note: dashed lines are 95% Hall percentile bootstrap confidence intervals, 2000 pseudo samples were used for bootstrapping.

Source: own estimations.

On the basis of SVAR, potential output and the output gap were estimated. It should be noted, that since VAR model has 5 lags we lose five observations at the beginning of the sample. Additionally, we estimated SVAR, where $\Delta rgdp_t^{sa}$ and Δcpi_t^{sa} was corrected for structural breaks depicted on Figure 2 (the time series were de-meaned before estimation of the SVAR and while obtaining potential output, the means of the variables are returned). In accordance with a battery of tests, the model with 4 lags and without time trend is chosen. The null hypothesis of no serial correlation up to 4 lags and residual heteroskedasticity are not rejected at any convenient levels. As in the previous model, there is evidence of non-normality of residuals in the equation for Δcpi_t^{sa} and the residuals in the equation $\Delta rgdp_t^{sa}$ are normally distributed. The obtained results are presented on Figure 4.

Figure 4. Potential output and output gap: SVAR (log scale)



Source: own estimations.

As one can see from Figure 4, output gaps obtained within SVARs are very sensitive to model specification and thus can differ drastically. However, it should be noted that the output gap from SVAR5 is very close to that presented by the NBB (2014)[12] at least for the period 2009q1–2013q4 both in terms of turning points and magnitudes. At the same time, it differs substantially from that presented in the country reports for Belarus of the IMF (2013).

5.2. Unobserved component model

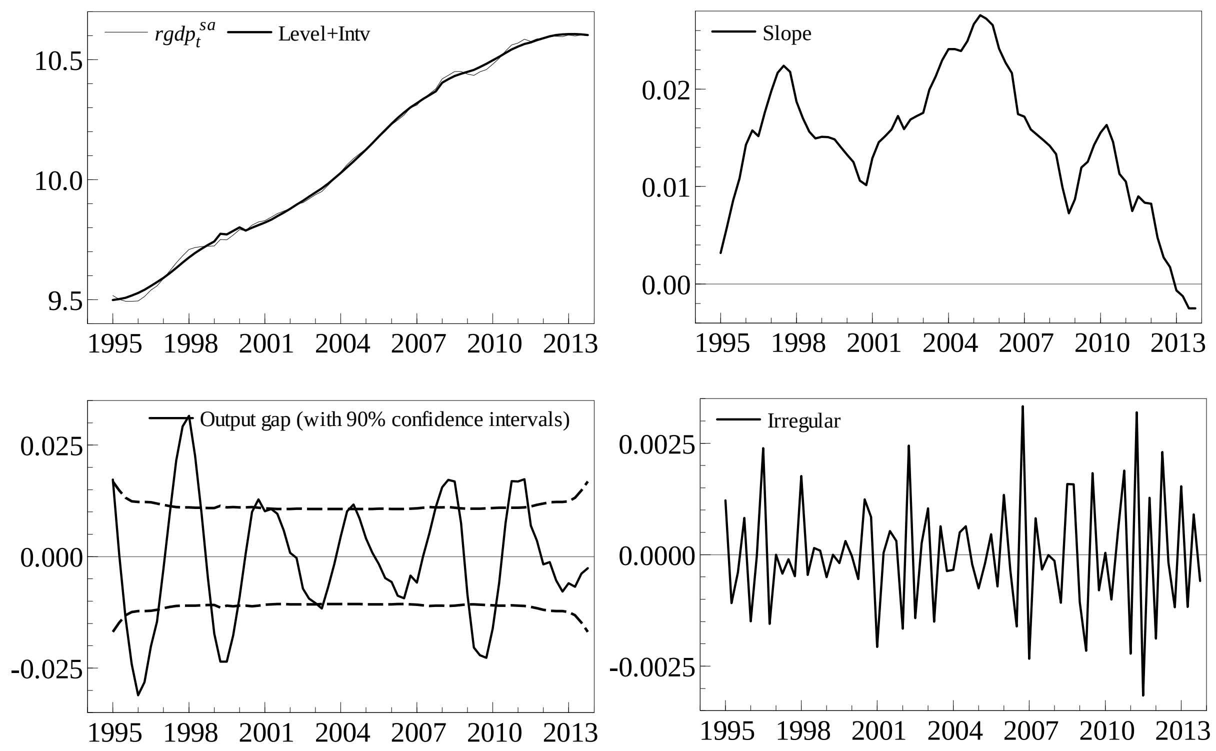
To get the potential output and output gap from the UC model, we at first follow Harvey (2011) in utilizing a univariate unobserved component model. There are several possible specifications of such a model and we used a so-called smooth trend model where the level (trend) is fixed and the slope (growth rate of the trend) is stochastic. Since we work with seasonally adjusted data, a seasonal component is excluded from the model. Additionally, the model contains a stochastic cycle of order one and an irregular component[13]. The model is corrected for possible structural breaks using an automatic detection procedure implemented in STAMP 8.3 (see section 3.2). The results are presented in Figure 5.

The output gap is depicted in Figure 5 with 90% confidence bands that permit to evaluate its significance at different peaks. As one can see from the graph, during the last period the output gap from the univariate UC model was negative but insignificant. Moreover, in 2013 it tended to be closed. The slope indicates the growth rates of potential output that has a clear-cut tendency to drift downwards, even below zero at the end of the sample (2013).

Hereafter, we included an inflation variable into the UC model (like in SVAR) and estimated potential output and the output gap in a bivariate context. In fact, a bivariate UC model that contains inflation and real GDP can be considered as a special type of the Philips curve model.

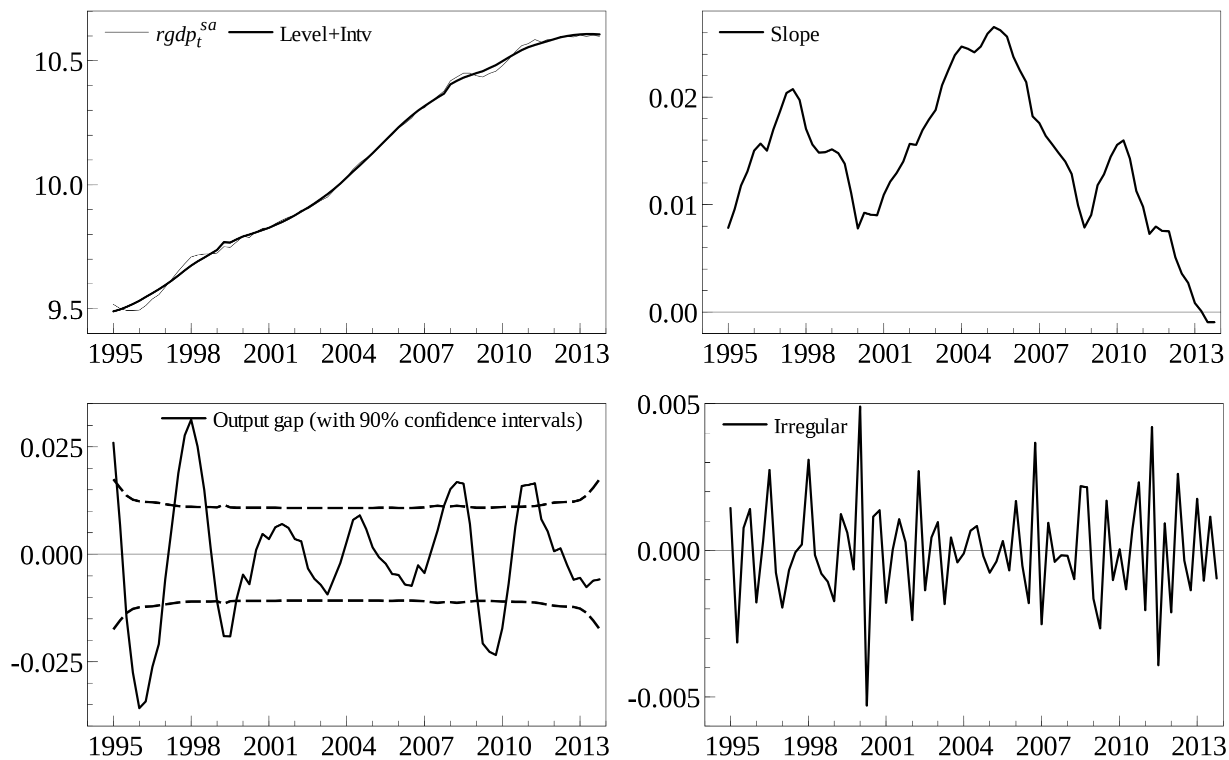
[12] The output gap presented in NBB (2014) is derived using DSGE model.
[13] Higher orders of the cycle can also be used, that will lead to a smoother cycle; see Harvey, Trimbur (2003).

Figure 5. Potential output and output gap: univariate UC (log scale)



Source: own estimations.

Figure 6. Potential output and output gap: multivariate UC (log scale)



Source: own estimations.

This model permits not only to obtain the output gap, taking into account the dynamics of inflation, but also to verify the existence of the relationship between the output gap and inflation (in accordance with (18–19)). In the system of equations we used the same specification for real GDP as it was discussed above in the univariate case (fixed level and stochastic slope) and include dummies that take structural breaks into account. The equation for inflation is a so-called local level model (stochastic level and no slope). It also contains dummies characterizing level shifts.

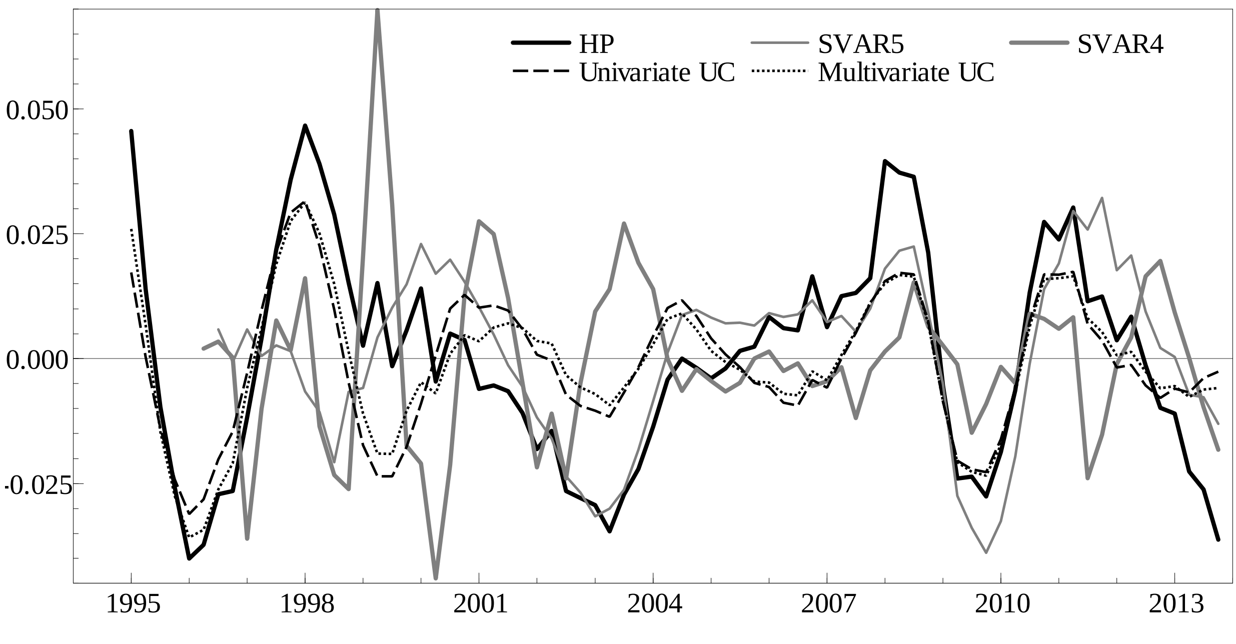
The potential output and output gap from the multivariate UC models are shown in Figure 6. There are some minor differences in comparison with derived unobservebles from the univariate model, but in general the output gaps in Figures 6 and 5 look similar. It is important to note that in the multivariate case we are able to estimate the coefficient β of the output gap variable in inflation equation (19).[14] If correlation between cycles of real GDP and inflation in the multivariate UC model are nearly perfect (in our case the correlation coefficient is equal to 1), one can use expression (18) and the cycle variance matrix, obtained from multivariate UC model to get the value of the this coefficient, that is equal to 1.89. Statistical significance of the coefficient will be evaluated later within the univariate UC model of inflation with the output gap as an explanatory variable. The multivariate UC model verifies the existence of a Philips curve-like relationship between inflation and the output gap in Belarus. This relationship can be obtained only if the structural breaks in inflation dynamics are taken into account. Otherwise, there is no evidence of a positive link between inflation and output gap within multivariate UC model.

In the next section we use the output gaps from univariate and multivariate UC models in UC model of inflation to evaluate the information content of the output gap while predicting inflation in Belarus.

5.3. Comparison of methods

In order to compare the output gaps obtained from different approaches we show them jointly in Figure 7. The output gap based on the HP-filter is added as a benchmark for comparison. Additionally, the correlation matrix of the different output gaps are calculated and presented in Table 2.

Figure 7. Different measures of the output gap (log scale)



Source: own estimations.

[14] See Weber (2011) for a method identifying simultaneous effects in UC models.

As follows from Figure 7 and Table 2, the output gaps obtained by different methods differ considerably in terms of their profiles and correlations with each other. It should be noted immediately that the output gap obtained using SVAR with corrections for structural breaks (SVAR4) differs cardinally from all other measures of the output gap and in fact does not correlate with them. The correlations with other output gaps are in fact negligible. The benchmark output gap estimated on the basis of HP-filter has rather strong correlations with SVAR4, univariate and multivariate UC (0.61–0.74). The output gap, derived from SVAR5 is rather weakly correlated with the output gaps obtained by univariate and multivariate UC models (0.4 and 0.37 respectively). On the contrary, the output gaps from UC models are closely correlated (0.97) and have similar profiles.

Table 2. Different measures of the output gap: correlation matrix

Output gap	HP	SVAR5	SVAR4	Univariate UC	Multivariate UC
HP	1				
SVAR5	0.608	1			
SVAR4	0.064	0.001	1		
Univariate UC	0.663	0.399	−0.051	1	
Multivariate UC	0.735	0.373	−0.038	0.965	1

Note: sample 1996q3–2013q4 is used to make estimates of the different output gaps comparable.

Source: own estimations.

Thus, the different methods of output gap estimation lead to quite distinct results. Three of them (HP, SVAR4, SVAR5) are clearly negative at the end of the sample without any tendency to be closed. The output gap, estimated using SVAR5 seems very similar to those obtained by the NBB, at least for 2009q1–2013q4. The output gaps derived from UC models are also negative at the end of the sample but with clear-cut tendency to closing.

5.4. Output gap as an indicator of inflation

In this section we evaluated the usefulness of different measures of the output gap derived above in predicting inflation. It must be emphasized that our task here is not to get well specified models of inflation, taking into account all possible determinants. On the contrary, we used very simple models of inflation where the output gap is the only explanatory variable in order to determine the sign and test the significance of this variable and to verify its information content in explaining inflation dynamics. The following consideration is behind such kind of analysis: If the output gap is clearly not significant or has a wrong sign in a simple model of inflation, it is rather improbable that the situation will change fundamentally in more complex models.

The modelling strategy is as follows: (1) for all measures of the output gap (HP, SVAR5, SVAR4, univariate UC, multivariate UC) the models with the output gap and inflation with lag 1 (to capture inflation inertia) and a constant were used. Additionally, step dummies characterizing the mean shifts of inflation are also included into the models.[15] The models are estimated using OLS; (2) univariate models of inflation with fixed level and no slope was applied for the output gaps derived from univariate and multivariate UC model. As in the previous case, appropriate step dummies have been included as well.[16] The models are estimated using the Kalman filter.

The output gaps are included into the modes with lags 0, 1 and 2. The results obtained are presented in Table 3 and 4. As one can see from Table 3, there is very weak evidence of the relationship between output gap and inflation if the output gaps based on HP, SVAR5 and SVAR4 are considered. In the most cases the estimated coefficients of the output gap are insignificant or have the wrong signs. Only the output gaps derived from HP-filter and SVAR4 with 2 lags are significant at 10% level and demonstrate a positive link with inflation. In sum, one can conclude that these output gaps are not reliable indicator variables for inflation.

---

[15] The same step dummies were used in all models. The following dummies correcting for mean shifts were included: s1998q4, s2000q2, s2002q3, s2011q2, s2012q1 (see section 2), and all of them are significant in the appropriate regressions.

[16] Only significant step dummies were retained in the models, so s2002q3 was excluded.

Table 3. Testing output gaps coefficients in AR(1) models of inflation

Lag	Output gap (HP)		Output gap (SVAR5)		Output gap (SVAR4)		Output gap (univariate UC)		Output gap (multivariate UC)	
	Without structural breaks									
	coefficient	p-value	coefficient	p-value	coefficient	p-value	coefficient	p-value	coefficient	p-value
0	0.521	0.153	0.180	0.707	−1.262	0.004	0.271	0.636	0.624	0.276
1	0.536	0.154	−0.412	0.392	−0.096	0.833	0.495	0.379	0.547	0.332
2	0.643	0.089	−0.180	0.725	0.877	0.054	0.637	0.248	0.563	0.310
	With structural breaks									
	coefficient	p-value	coefficient	p-value	coefficient	p-value	coefficient	p-value	coefficient	p-value
0	0.023	0.920	−0.292	0.360	−1.094	0.000	0.423	0.250	0.466	0.190
1	0.167	0.474	−0.397	0.224	−1.127	0.000	0.641	0.073	0.651	0.062
2	0.320	0.173	−0.238	0.477	−0.560	0.079	0.652	0.058	0.689	0.043

Source: own estimations.

For the output gaps derived from univariate and multivariate UC model the situation is somewhat different. Although in the AR(1) without structural breaks models all these gaps are insignificant, models with structural breaks demonstrate positive and significant links between inflation and the output gap with lag 1 and 2 (at 10% and 5% significance level respectively). For the output gap obtained from univariate UC model the appropriate coefficients are equal to 0.64 and 0.56 accordingly; for the output gap obtained from multivariate UC model the coefficients are to 0.65 and 0.69. Thus, the output gaps derived from univariate and multivariate UC models seem to be more relevant when considering the relationship between output gap and inflation.

Table 4. Testing output gaps coefficients in the inflation models: UC model

Lag	Univariate UC		Multivariate UC	
	Without structural breaks			
	coefficient	p-value	coefficient	p-value
0	0.078	0.954	2.313	0.104
1	0.820	0.527	1.438	0.232
2	1.100	0.319	0.958	0.235
	With structural breaks			
	coefficient	p-value	coefficient	p-value
0	1.290	0.032	2.025	0.001
1	1.762	0.029	1.574	0.000
2	0.824	0.021	0.740	0.039

Source: own estimations.

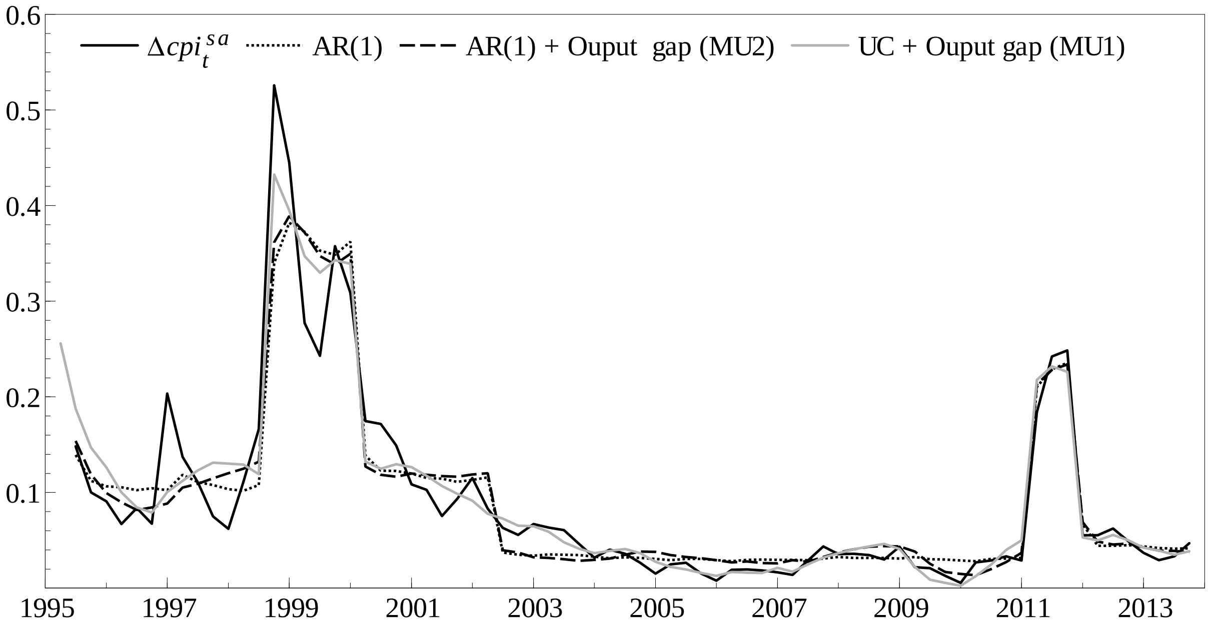
In contrast to the previous results, output gaps obtained from UC models in all cases demonstrate positive links with inflation (Table 4). When structural breaks in the mean of inflation are not taken into account, these links still remain statistically insignificant. However, if the UC model of inflation is estimated with appropriate step dummies, the situation is to change considerably. For all lags from 0 to 2 the output gaps appeared to be statistically significant at 1% or 5% levels. The values of the coefficients of the output gap are varying, depending on the lag chosen in the model, from 0.74 to 2.03. Thus, the UC model of inflation allows us identifying a positive and significant relationship between output gap and inflation in Belarus.[17] The results from a univariate model of inflation are not contradicting those obtained within a multivariate UC model where inflation and real GDP are considered simultaneously. In general, our results are in line with Harvey (2011), where a positive and statistically significant linkage between the output gap and inflation was determined with univariate and multivariate UC models. However, the output gaps derived from UC models in our case are also relevant to some

[17] It is interesting to note that the output gap derived using HP-filter is also statistically significant in UC models of inflation with lags 1 and 2 and demonstrates the right sign.

extent in AR(1) models of inflation, while in Harvey (2011) the autoregressive model of inflation with the output gap as explanatory variable leads to estimates that are erratic and difficult to interpret.

The graphical illustration of the goodness of fit for the autoregression model with constant and the appropriate step dummies in comparison with the model with the most significant coefficients of the output gaps (see Table 3 and 4) is given in Figure 8.

Figure 8. Actual and fitted values of inflation



Source: own estimations.

In order to evaluate predictive performance of the models without and with the output gap we compared the models having significant coefficients at the output gap, presented in Table 3, with the benchmark model that comprises inflation with lag 1, constant and appropriate step dummies. Then, the models from Table 4 were compared with the benchmark model and with the model that has the most significant coefficient at the output gap. To this end, the pseudo out-of-sample 1-step forecasts were estimated for the period 2009q1–2013q4 (20 quarters).

Since in the first case all the models are nested, the test for equal forecast accuracy proposed by Clark and McCracken (2001) is appropriate. This procedure generates pseudo out-of-sample forecasts, estimates forecast errors and tests for equality of mean square error (MSE) and encompassing for each pair of nested models, where the first model is a restricted version of the second model. To compare forecasts two statistics are used, MSE-*t* and MSE-*F.* The hypothesis is rejected if MSE-*t* and MSE-*F* exceed the appropriate critical values. In other words, we test the null hypothesis that the unrestricted model (with the output gap) forecast MSE is equal to the restricted model (without the output gap) forecast MSE against the one-sided upper tail alternative hypothesis that the unrestricted model forecast MSE is less than the restricted model forecast MSE.

Further statistics in Clark-McCracken test are ENC-*t* and ENC-*F* which relate to the concept of forecast encompassing. In this context, if the restricted model forecast (without the output gap) encompass the unrestricted model forecast (with the output gap), the output gap provides no useful additional information for prediction inflation. If forecast encompassing is rejected, then the output gap does contain information content useful for predicting dynamics of inflation. Performing Clark-McCracken tests for equal forecast accuracy the recursive scheme is applied, where forecasting models estimated with more data as forecasting moves forward in time. The results are presented in Table 5.

As follows from the results obtained, MSEs for all models with the output gap are less than MSE for the benchmark model of inflation. Formal testing the differences between the mean square errors shows that the null hull hypotheses that models forecasts MSEs with the output gaps are equal to the models forecasts MSEs without the output gaps are rejected for all considered models at 1% or 5% significance levels. The same situations one can see for forecast encompassing. The null hypotheses are also rejected for all unrestricted models at 1% or 5% significance levels. Therefore, we can conclude that the models of inflation with the output gaps outperform the models without the output gaps in pseudo out-of sample exercises.

Table 5. Clark-McCracken tests for equal forecast accuracy (out-of-sample forecast for 2009q1–2013q4)

Model	MSE	MSE-*t*	MSE-*F*	ENC-*t*	ENC-*F*
AR(1)with constant and step dummies (benchmark) vs.:	0.00462	-	-	-	-
AR(1)with constant and step dummies + output gap (UUC1)	0.00396	1.783	3.227	1.890	1.963
AR(1)with constant and step dummies + output gap (UUC2)	0.00388	1.855	3.787	2.025	2.135
AR(1)with constant and step dummies + output gap (MUC1)	0.00406	1.843	2.756	2.060	1.577
AR(1)with constant and step dummies + output gap (MUC2)	0.00404	1.975	2.867	2.224	1.643
		Critical values			
1%		1.659	2.631	2.004	1.948
5%		1.019	1.242	1.343	1.007

Note: we experimented with the other out-of-sample forecast periods (less and more than 20 quarters) and the general conclusions remain the same. UUC1 and UUC2 are the output gaps derived from the univariate UC models with lags 1 and 2 respectively.

Source: own estimations.

To evaluate pseudo out-of sample performance of the UC models presented in Table 4, in comparison with two benchmark models (the first one is AR(1) model with constant and the appropriate step dummies and the second one is the model from Table 3 with the most significant coefficient at the output gap), we employed Diebold and Mariano test (1995) for equal forecast accuracy suited for non-nested models. Diebold-Mariano utilizes the actual time series of inflation and pairs of competing predictions, applying a loss criterion (MSE in our case) and calculating a measure of predictive accuracy that allow the null hypothesis of equal forecasts accuracy to be tested. The S(1) statistic tests the mean difference between the loss criteria for two predictions is zero, using a long-run estimate of the variance of the difference time series. If the null hypothesis is rejected, than pseudo out-of-sample forecast from competing model outperform the forecast from the benchmark model.

Table 6. Diebold-Mariano tests for equal forecast accuracy (out-of-sample forecast for 2009q1–2013q4)

Diebold-Mariano test	UUC0	UUC1	UUC1	MUC0	MUC1	MUC1
	vs. AR(1)with constant and step dummies					
MSE(tested)−MSE (benchmark)	0.00372	0.00393	0.00403	0.00356	0.00385	0.0040
S(1)	2.499	2.537	2.782	2.193	2.567	2.777
p-value	0.0143	0.0112	0.0054	0.0283	0.0010	0.0055
	vs. AR(1)with constant and step dummies + output gap (MUC2)					
MSE(tested)−MSE (benchmark)	0.00334	0.00355	0.00365	0.00318	0.00347	0.00366
S(1)	2.417	2.520	2.794	2.136	2.552	2.787
p-value	0.0156	0.0117	0.0052	0.0327	0.0107	0.0053

Note: maximum order of the lag using in calculating the long-run variance of the difference series from its autocovariance function calculated from the Schwert criterion as a function of the sample size. The uniform kernel is employed in calculations. UUC0, UUC1 and UUC2 are the output gaps derived from the univariate UC models with lags 0, 1 and 2 respectively. MUC0, MUC1 and MUC2 are the output gaps derived from the multivariate UC models with lags 0, 1 and 2 respectively.

Source: own estimations.

The results of Diebold-Mariano tests for equal forecast accuracy are presented in Table 6. As one can see from the obtained results, all UC models of inflation with the output gaps derived from both univariate and multivariate UC models outperform the benchmark models in pseudo out-of sample forecasts at 1% or 5% significance levels. These results lead to conclusion that the output gaps extracted in UC models have the most information content while predicting inflation dynamics.

6. Conclusions and policy implications

The existing linkage between real economy and inflation is traditionally of great importance for monetary authorities. In this context, the concept of the output gap plays a prominent role in conventional macroeconomic theory, applied research and monetary policy analysis. In this study, we used structural vector autoregression and unobserved component models to obtain new measures of the output gap in Belarus, and tried to relate them to inflation. The output gap, based on a HP-filter approach, is considered as a benchmark. The following conclusions can be drawn from our research:

1. The estimates of the output gaps using different methods demonstrate quite different results. Nevertheless, all of them are negative at the end of the sample. The output gap estimated on the basis of a SVAR model (without structural breaks in the variables taken into account) is very close to the output gap obtained by the NBB (2014) at least for the period 2009q1–2013q4. SVAR-based output gaps appeared to be very sensitive to a model specification and thereby can differ substantially.

2. The output gaps derived from univariate and multivariate UC models remain negative at the end of the sample, but have a clear-cut tendency to be closed. Moreover, these output gaps are statistically insignificant at the end of the sample. In accordance with conventional macroeconomic theory, this means that current monetary policy in Belarus should not be softened.

3. There is no statistically significant relationship between different measures of the output gap and inflation if the existent structural breaks in inflation dynamics are taken into account. Aside from that, in several cases the coefficients at the output gap have the wrong (negative) sign. If these structural breaks are included into the model by means of appropriate step dummies, the situation changes dramatically for UC models of inflation. The output gaps derived from the univariate and multivariate UC model are borderline significant in the AR(1) with constant and the appropriate step dummies, estimated using OLS. At the same time, OLS estimates of the output gaps derived from the HP-filter and SVAR models do not give meaningful results even if structural breaks are included into the models.

4. A multivariate UC model where real GDP, inflation and the output gap are estimated simultaneously, verifies the positive link between inflation and the output gap. The estimated coefficient is equal to 1.89. The univariate models of inflation confirm this finding and demonstrate that output gaps estimated on the basis of univariate and multivariate UC model are statistically significant and have a positive sign. The values of the coefficients of the output gap are varying, depending on the lag chosen in the model from 0.74 to 2.03. These results are novel for the Belarusian economy and, on our view, demonstrate the usefulness of the unobserved component models while modeling the relationship between the output gap and inflation.

5. Output gaps estimated on the basis of the UC models have definite information content in modeling inflation. UC models of inflation with the output gap outperform benchmark autoregressive models of inflation in pseudo out-of-sample forecasts.

6. At the same time it should be noted that the relationship between the output gap and inflation in Belarus seems not so obvious. Only the implementation of a special technique with multiple structural breaks taken into account leads to sensible results. In our view, there are unobservables, like the monetary overhang or the real money gap that will be used as more pronounced measures of inflation pressure in Belarus. Thus, the estimations of the monetary overhang or real money gap and modeling inflation within the P-star framework seem an interesting avenue for further research.

References

Bai, J., Perron, P. (1998). Estimating and testing linear models with multiple structural changes, *Econometrica*, 66, 47–78.

Bai, J., Perron, P. (2003). Computation and analysis of multiple structural change models, *Journal of Applied Econometrics*, 18, 1–22.

Billmeier, A. (2004). Ghostbusting: which output gap measures really matters? IMF Working Paper, WP/04/146.

Billmeier, A. (2009). Ghostbusting: which output gap really matters? *International Economics and Economic Policy*, 6, 391–419.

Blanchard, O. J., Quah, D. (1989). The dynamic effects of aggregate demand and supply disturbances, *American Economic Review*, 79, 655–673.

Clark, T. E., Michael W. McCracken, M. W. (2001). Tests of Equal Forecast Accuracy and Encompassing for Nested Models, *Journal of Econometrics*, 105, 85–110.

Cotis, J-P., Elmeskov, J., Mourougane, A. (2004). Estimates of potential output: benefits and pitfalls from a policy perspective. In *The euro area business cycles: stylized facts and measurement issues*. London: Centre for Economic Policy Research, 35–60.

Demidenko, M., Kuznetsov, A. (2012). Economic growth in Republic of Belarus: factors and estimations of equilibrium, Bankovskiy Vestnik, 26 (special issue), 1–60 (in Russian).

Diebold, F., Mariano, R. (1995).Comparing Predictive Accuracy, Journal of Business and Economic Statistics, 13, 253–263.

Elliot, G., Rothenberg, T.J., Stock, J.H. (1996). Efficient tests for an autoregressive unit root, *Econometrica*, 64, 813–836.

Ericsson, N., MacKinnon, J. (2002) Distributions of Error Correction Tests for Cointegration, *Econometrics Journal*, 5, 285–318.

Franses, P. H., Hobijn, B. (1997). Critical values for unit root tests in seasonal time series, *Journal of Applied Statistics*, 24, 25–48.

Harvey, A. (2011). Modelling the Phillips curve with unobserved components, *Applied Financial Economics*, 21, 7–17.

Harvey, A. C., Trimbur, T. M. (2003). General Model-Based Filters for Extracting Cycles and Trends in Economic Time Series, *The Review of Economics and Statistics*, 85, 244–255/

Hauptmeier, S., et al. (2009). *Projecting potential output: method and problems*. ZEW, Physica-Verlag Heidelberg.

Hodrick, R. J., Prescott, E. C. (1997). Postwar U.S. business cycles: an empirical investigation, *Journal of Money, Credit, and Banking*, 29, 1–16.

Hylleberg, S., Engle, R.F., Granger, C.W.J., Yoo. B.S. (1990). Seasonal integration and Cointegration, *Journal of Econometrics*, 44, 215–238.

IMF (2013). Republic of Belarus: 2013 Article IV Consultation and Fourth Post-Program Monitoring, *IMF Country Report*, 13/159.

IMF (2014). Republic of Belarus: fifth post-program monitoring discussions, *IMF Country Report*, 13/18.

Koopman, S. J., Harvey, A. C., Doornik, J. A., and Shephard, N. (2009). *STAMP 8.2: Structural time series analysis, modelling, and prediction using STAMP*. London, Timberlake Consultants Press.

Kruk, D. (2008). Forecasting inflation in the context of the transition to inflation targeting: the role of the output gap indicator. IPM Research Center, *Working paper*, WP/08/10 (in Russian).

Kwiatkowski, D. Phillips, P. C. B., Schmidt, P., Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics* 54,159–178.

Lim Choon Seeng, V. (2007) *Estimation and use of potential GDP for monetary policy for SEASEN countries*. SEASEN Research and Training Center, Kuala Lumpur, Malaysia.

Mironchik, N. (2006). Monetary analysis of the equilibrium and cycles: application of Kalman filter, *Bankovskiy Vestnik*, 26, 32–37 (in Russian).

Morley, J. (2010). The emperor has no clothes, *Macro Focus*, 5, June 24, 1–14.

NBB (2014). The Main Tendencies in Economy and Monetary Sphere in the Republic of Belarus, *Analytical Review of the National Bank of the Republic of Belarus*, 2013, Minsk.

Pelipas, I. (2011). Structural Breaks and Dynamic Characteristics of Inflation and Growth Rates of Monetary Aggregates, *BEROC Working paper* 015.

Pelipas, I. (2012). Multiple Structural Breaks and Inflation Persistence in Belarus, *BEROC Working paper* 021.

Pelipas, I. (2013) Paradoxes of seasonal adjustment, *Bankovskiy Vestnik*, 22, 22–31 (in Russian).

Tschernig, R., Weber, E., Weigand, R. (2013). Long-Run Identification in a Fractionally Integrated System. *Journal of Business and Economic Statistics*, 31, 438–450.

Tschernig, R., Weber, E., Weigand, R. (2014). Long- versus Medium-Run Identification in Fractionally Integrated VAR Models, *Economics Letters*, 122, 299–302.

Vetlov, I., Hledik, T., Jonsson, M., Kucsera, H., Pisan, M. (2011). Potential output in DSGE models, *ECB Working Paper*, 1351.

Weber, E. (2011). Analysing US Output and the Great Moderation by Simultaneous Unobserved Components, *Journal of Money, Credit and Banking*, 43, 1543–1751.

Annex. Data and testing for seasonal unit root test and multiple structural breaks

Table A1. HEGY seasonal unit root test

Variable	Model specification: Deterministic terms	Model specification: Number of lags	H_0	Test	Test statistics	Critical values: 1%	Critical values: 5%
			1995q1–2013q4				
rgdp	Constant, Trend, Seasonals	1	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−1.16	−3.96	−3.39
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−2.19	−3.41	−2.82
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	1.29	8.79	6.55
	Constant, Trend	1	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−1.28	−3.98	−3.40
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−2.18	−2.53	−1.93
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	0.52	4.76	3.05
Δ_4*rgdp*	Constant, Seasonals	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−2.93	−3.41	−2.84
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−4.43	−3.41	−2.83
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	36.87	8.79	6.57
	Constant	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−2.97	−3.42	−2.85
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−4.47	−2.53	−1.93
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	38.64	4.83	3.08
			1995q1–2013q4				
cpi	Constant, Trend, Seasonals	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−1.76	−3.96	−3.39
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−7.16	−3.41	−2.82
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	24.09	8.79	6.55
	Constant, Trend	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−1.74	−3.98	−3.40
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−8.37	−2.53	−1.93
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	18.96	4.76	3.05
			2001q1–2010q4				
cpi	Constant, Trend, Seasonals	1	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−1.91	−3.96	−3.39
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−2.56	−3.41	−2.82
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	8.48	8.79	6.55
	Constant, Trend	1	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−2.50	−3.98	−3.40
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−2.51	−2.53	−1.93
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	5.14	4.76	3.05
Δ_4*cpi*	Constant, Seasonals	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−2.36	−3.41	−2.84
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−3.53	−3.41	−2.83
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	12.66	8.79	6.57
	Constant	0	$H_0: \pi_1 = 0$	$t_{\pi 1}$	−3.71	−3.42	−2.85
			$H_0: \pi_2 = 0$	$t_{\pi 2}$	−4.47	−2.53	−1.93
			$H_0: \pi_3 = \pi_4 = 0$	F_{34}	13.88	4.83	3.08

Note: The following regression is used to conduct the HEGY seasonal unit root test:

$$\Delta_4 y_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-2} + \pi_4 z_{3,t-1} + \sum_{j=1}^{p-4} \alpha_i \Delta_4 y_{t-j} + \varepsilon_t ,$$

where $\Delta_4 y = (1-L^4) = y_t - y_{t-4}$; $z_{1,t} = (1+L+L^2+L^3)y_t$; $z_{2,t} = (-1+L+L^2+L^3)y_t$ $z_{3,t} = (-1-L^2)y_t$; with L being the lag operator; ε_t are residuals. The deterministic terms such as constant, trend and seasonal dummies can be added to the regression. The number of lagged seasonal differences is chosen to eliminate residual autocorrelation. The null hypothesis $H_0 : \pi_1 = 0$, $H_0 : \pi_2 = 0$ and $H_0 : \pi_3 = \pi_4 = 0$ corresponds to test for regular, semiannual and annual unit root, respectively. The regression is estimated by OLS and the hypotheses are tested using corresponding *t*-test for the first two hypotheses $(t_{\pi 1}, t_{\pi 2})$ and *F*-test for the third one (F_{34}). The critical values are taken from Franses, Hobijn (1997). Rejections of the null hypotheses are marked in gray.

Source: own estimations.

Table A2. Bai-Perron multiple breakpoint test for inflation

Multiple breakpoint test
Compare information criteria for 0 to M globally determined breaks
Sample: 1995Q1 2013Q4
Included observations: 74
Breakpoint variables: Constant
Break test options: Trimming 0.05, Max. breaks 5

Schwarz criterion selected breaks: 5
LWZ criterion selected breaks: 5

Breaks	# of Coefs.	Sum of Sq. Resids.	Log-L	Schwarz* Criterion	LWZ* Criterion
0	1	0.825115	62.69413	-4.45215	-4.41028
1	3	0.505882	81.03987	-4.82624	-4.70006
2	5	0.315410	98.75600	-5.18354	-4.97228
3	7	0.275436	103.83790	-5.20392	-4.90677
4	9	0.227250	111.04940	-5.28110	-4.89719
5	11	0.171926	121.51130	-5.44495	-4.97336

* Minimum information criterion values displayed with shading

Estimated break dates:
1: 2001Q1
2: 1998Q4, 2000Q2
3: 1998Q4, 2000Q2, 2002Q2
4: 1998Q4, 2000Q2, 2011Q2, 2012Q1
5: 1998Q4, 2000Q2, 2002Q3, 2011Q2, 2012Q1

Source: own estimations using Eviews 8 econometric software.

Table A3. Bai-Perron multiple breakpoint test real GDP growth

Multiple breakpoint tests
Compare information criteria for 0 to M globally determined breaks
Sample: 1995Q2 2013Q4
Included observations: 75
Breakpoint variables: Constant
Break test options: Trimming 0.05, Max. breaks 5

Schwarz criterion selected breaks: 5
LWZ criterion selected breaks: 2

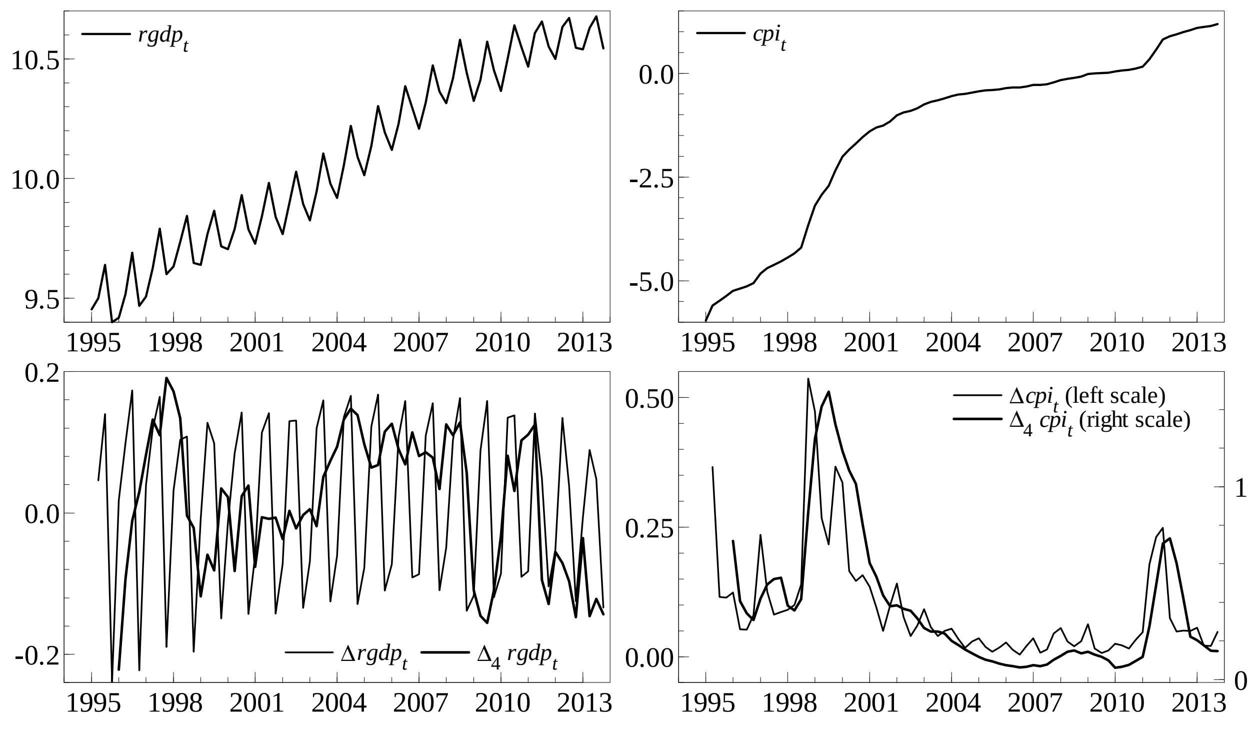
Breaks	# of Coefs.	Sum of Sq. Resids.	Log-L	Schwarz* Criterion	LWZ* Criterion
0	1	0.011754	222.1199	-8.70351	-8.66163
1	3	0.009801	228.9317	-8.77002	-8.64384
2	5	0.007718	237.8928	-8.89385	-8.68260
3	7	0.006988	241.6212	-8.87814	-8.58100
4	9	0.005728	249.0776	-8.96185	-8.57794
5	11	0.005004	254.1429	-8.98179	-8.51021

* Minimum information criterion values displayed with shading

Estimated break dates:
1: 2011Q3
2: 1996Q2, 2011Q3
3: 1996Q2, 1998Q2, 2011Q3
4: 1996Q2, 1998Q2, 2003Q3, 2008Q2
5: 1996Q2, 1998Q2, 2003Q3, 2008Q2, 2011Q3

Source: own estimations using Eviews 8 econometric software.

Figure A1. Data used (log scale, seasonally unadjusted)



Source: own estimations based on Belstat data.